



BADSWORTH
CofE School

CARE – Prepare – Believe

'I can do all things through him who strengthens me'

Philippians 4:13

Calculation Policy

This policy is a working document and will be revised and amended as necessary. Some images and wording have been taken from White Rose Maths Calculation Policy and NCETM PD Materials. Other parts are from knowledge gained on Maths Hub Subject Leader Training. It is linked to our Maths Curriculum policy where more details can be found to support this policy.

Introduction

This document sets out to show a clear progression of how calculations are taught at Badsworth CE J&I School. It is broken down into the four operations. There is also an additional section on times tables that links to the multiplication section and an additional section addressing the Early Years Counting Principles. For each year group, a key objective is listed that will be taught, followed by some examples of how this will be taught in that year group. The examples are broken down further into 3 key sections: concrete, pictorial and abstract with pictures to help you understand how to teach the concept best. There are also several appendices that are crucial to supporting calculations:

- Appendix 1: Glossary
 - o Here you will find meanings for many of the new words introduced in this policy.
- Appendix 2: Written Method Scripts
 - o Here you will find the exact wording for how children are taught to structure and calculate using formal written methods.
- Appendix 3: Additional Questions for General Mathematical Talk
 - o Here you will find a table with key questions that are non-year group specific and to be used across school.
- Appendix 4: More information on resources, models and representations
 - o Here you will find more detailed explanations of each of the examples you will find in the main policy. This is also broken down into the four operations and different representations, models and resources best support teaching and learning.

Concrete, Pictorial, Abstract Approach (CPA)

As our school follows a teaching for mastery approach, the policy heavily relies on the CPA approach. This approach helps children to gain a deeper understanding of the concept they are being taught and allows all children to access the content at their own level. When moving stages using this approach, the previous stage should be shown alongside e.g. concrete should be shown alongside pictorial, pictorial alongside concrete. Movement between the 3 stages should be fluid and teachers/parents should be flexible about moving their child back and forth along the stages to best suit their learning needs.

Differentiation

Mastery works on the principle that most children will progress through the curriculum at the same pace, staying on the same topic. The White Rose Maths Scheme that we follow provides 'low floor, high ceiling' tasks which will suit most children within a given year group with the idea that they will keep up instead of having to catch up. The small steps allow teachers to identify gaps in children's knowledge and this is then used to provide support where children need it. Teachers provide targeted support, remove barriers and provide additional resources for children who are struggling with a particular concept, without the need for separate worksheets or tasks. This may include pupils with SEND who may access material or structures from earlier year groups as appropriate to their particular needs. Furthermore, various reasoning and problem-solving questions are used that will provide challenge for pupils who already understand new concepts quickly and require a greater depth of understanding within the same concept.

Regrouping

In many of the examples in the policy below, you will see regrouping in a number of calculations. It is important to remember, children **must** be exposed to the new concept (e.g. addition of two 2-digit numbers) via examples without regrouping initially before regrouping can be introduced. Another key point to remember is that even when a question does not require regrouping, the scripts in Appendix 2 should still be followed as children must be asked 'Do I need to make an exchange?' Too often teachers/parents get into the habit of only asking if they need to make an exchange when they actually do need to make an exchange. By asking this question all the time, children become more proficient at recognising for themselves when an exchange is necessary.

Varied Fluency

Children should be given a variety of questions structured in as many ways as possible for them to become fluent and embed the key concept they are learning about. To become fluent, children must be efficient, accurate and flexible in their calculations. By varying the question styles, children are able to recognise the maths behind the problem more easily and become more resilient with their maths in general. One great way to do this is to have them complete questions about the four calculations in the context of different measures e.g. capacity, length, weight, money for decimals etc.

Reasoning and Problem Solving

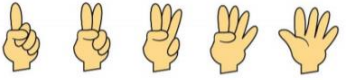



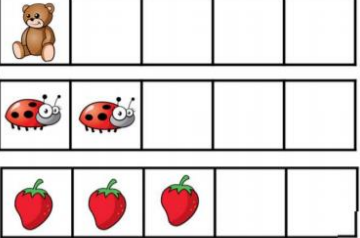
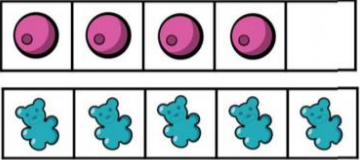

Previously, only children thought to be "higher ability" were exposed to reasoning and problem-solving style questions. However, the mastery approach ensures all children are exposed to these types of questions to help them develop these skills at their own level. By doing this, we deepen children's thinking and provide challenge for those children who need it.

Mathematical Talk

Mathematical talk is a key component when using a teaching for mastery style. Children should be given plenty of opportunities to discuss maths, use the key vocabulary for that unit of work and answer questions to deepen their thinking. When answering a question, children should be encouraged to answer in full sentences. These sentences can then be repeated by small groups or whole class to embed learning further. This particularly support learners who are less confident as they are getting to hear the language being used correctly. Children should be given stem sentences to help them structure their answers. This is where parts of a sentences are given as some examples of these can be seen throughout the policy. Key vocabulary for each topic and year group is also given at the top of each section in the policy.

Questioning is also crucial to develop learning. Some questions that relate to these key concepts are given with the key vocabulary at the start of each section in the policy. Remember to refer to the 'mathematical talk' section of our WRM planning scheme for further questions that will be relevant to the specific objective you are teaching. As well as more specific questions, there are many general questions that can be used across all year groups during maths sessions. These relate to fluency, reasoning and problem-solving work. These general questions for all can be found in the table in **Appendix 3**.

Skills and representations overview by Year Group

<p><u>Early Learning Goals - Children at the expected level of development will:</u></p>	<p><u>Key Representations and Principles</u></p>	
<p>Have a deep understanding of number to 10 including the composition of each number</p>	<p>1</p> <p>The one-one principle. This involves children assigning one number name to each object that is being counted. Children need to ensure that they count each object only once ensuring they have counted every object.</p>	 
<p>Subitise (recognise quantities without counting) up to 5</p>	<p>2</p> <p>The stable-order principle. Children understand when counting, the numbers have to be said in a certain order.</p>	
<p>Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts</p>	<p>3</p> <p>The cardinal principle. Children understand that the number name assigned to the final object in a group is the total number of objects in that group.</p>	
<p>Verbally count beyond 20, recognising the pattern of the counting system</p>	<p>4</p> <p>The abstraction principle. This involves children understanding that anything can be counted including things that cannot be touched including sounds and movements e.g. jumps.</p>	
<p>Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity</p>	<p>5</p> <p>The order-irrelevance principle. This involves children understanding that the order we count a group of objects is irrelevant. There will still be the same number.</p>	
<p>Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally</p>		

Addition

<u>Year</u>	<u>Skill</u>	<u>Representations, Models and Strategies</u>	
1	Add 1-digit numbers to 10	Part-whole model Bar model Number shapes	Ten frames (within 10) Bead strings (10) Number tracks
1	Add 1 and 2-digit numbers to 20	Part-whole model Bar model Number shapes Ten frames (within 20)	Bead strings (20) Number tracks Number lines (labelled) Straws
2	Add three 1-digit numbers	Part-whole model Bar model	Number shapes Ten frames (within 20)
2	Add 1 and 2-digit numbers to 100	Part-whole model Bar model Number lines (labelled)	Number lines (blank) Straws Hundred squares
2	Add two 2-digit numbers	Part-whole model Bar model Number lines (blank) Straws	Base 10 Place value counters Column addition
3	Add with up to 3-digits	Part-whole model Bar model Base 10	Place value counters Column addition
4	Add with up to 4-digits	Part-whole model Bar model Base 10	Place value counters Column addition
5/6	Add with more than 4-digits	Part-whole model Bar model	Place value counters Column addition
5/6	Add with up to 3 decimal places	Part-whole model Bar model	Place value counters Column addition

Subtraction

<i>Year</i>	<i>Skill</i>	<i>Representations, Models and Strategies</i>	
1	Subtract two 1-digit numbers to 10	Part-whole model Bar model Number shapes	Ten frames (within 0) Bead strings (0) Number tracks
1	Subtract 1 and 2-digit numbers to 20	Part-whole model Bar model Number shapes Ten frames (within 20)	Bead strings (20) Number tracks Number lines (labelled) Straws
2	Subtract 1 and 2-digit numbers to 100	Part-whole model Bar model Number lines (labelled)	Number lines (blank) Straws Hundred square
2	Subtract two 2-digit numbers	Part-whole model Bar model Number lines (blank) Straws	Base 10 Place value counters Column addition
3	Subtract with up to 3-digits	Part-whole model Bar model Base 10	Place value counters Column addition
4	Subtract with up to 4-digits	Part-whole model Bar model Base 10	Place value counters Column addition
5/6	Subtract with more than 5-digits	Part-whole model Bar model	Place value counters Column addition
5/6	Subtract with up to 3 decimal places	Part-whole model Bar model	Place value counters Column addition

Multiplication

<u>Year</u>	<u>Skill</u>	<u>Representations, Models and Strategies</u>	
1/2	Solve one-step problems with multiplication	Bar model Number shapes Counters	Ten frames Bead strings Number lines
3/4	Multiply 2-digit by 1-digit numbers	Place value counters Base 10	Short written method Expanded written method
4	Multiply 3-digit by 1-digit numbers	Place value counters Base 10	Short written method
5	Multiply 4-digit by 1-digit numbers	Place value counters	Short written method
5	Multiply 2-digit by 2-digit numbers	Place value counters Base 10	Short written method Grid method
5	Multiply 2-digit by 3-digit numbers	Place value counters Short written method	Grid method
5/6	Multiply 2-digit by 4-digit numbers	Formal written method	

Multiplication - Times tables

Children will recall and use multiplication facts for the...

Year	Skill	Representations, Models and Strategies	
2	...2-times table	Bar models Number shapes Counters Money	Ten frames Bead strings Number lines Everyday objects
2	...5-times table	Bar models Number shapes Counters Money	Ten frames Bead strings Number lines Everyday objects
2	...10-times table	Bar models Number shapes Counters Money	Ten frames Bead strings Number lines Everyday objects
3	...3-times table	Hundred square Number shapes Counters	Bead strings Number lines Everyday objects
3	...4-times table	Hundred square Number shapes Counters	Bead strings Number lines Everyday objects
3	...8-times table	Hundred square Number shapes	Bead strings Number lines Everyday objects
4	...6-times table	Hundred square Number shapes	Bead strings Number lines
4	...7-times table	Hundred square Number shapes	Bead strings Number lines
4	...9-times table	Hundred square Number shapes	Bead strings Number lines
4	...11-times table	Hundred square Base 10	Place value counters Number lines
4	...12-times table	Hundred square Base 10	Place value counters Number lines




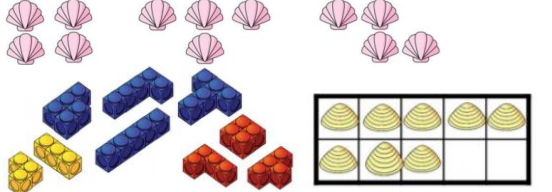
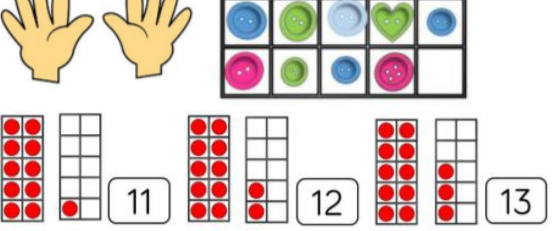
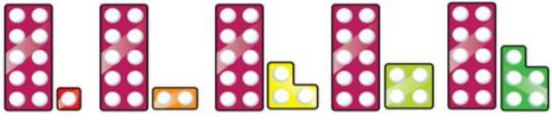

Division




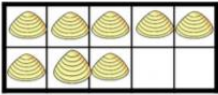
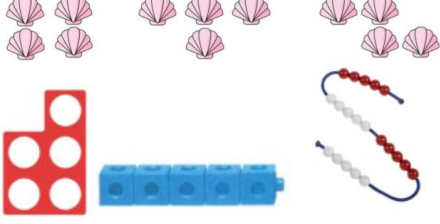
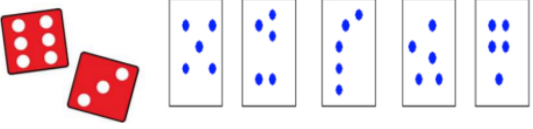
<i>Year</i>	<i>Skill</i>	<i>Representations, Models and Strategies</i>	
1/2	Solve one-step problems with division (sharing)	Bar model Real life objects	Arrays Counters
1/2	Solve one-step problems with division (grouping)	Real life objects Number shapes Bead strings Ten frames	Number lines Arrays Counters
3	Divide 2-digits by 1-digit (no exchange sharing)	Straws Base 10 Bar model	Place value counters Part-whole model
3	Divide 2-digits by 1-digit (sharing with exchange)	Straws Base 10 Bar model	Place value counters Part-whole model
3/4	Divide 2-digits by 1-digit (sharing with remainders)	Straws Base 10 Bar model	Place value counters Part-whole model
4/5	Divide 2-digits by 1-digit (grouping)	Place value counters Counters	Place value grid Written short division
4	Divide 3-digits by 1-digit (sharing with exchange)	Base 10 Bar model	Place value counters Part-whole model
4/5	Divide 3-digits by 1-digit (grouping)	Place value counters Counters	Place value grid Written short division
5	Divide 4-digits by 1-digit (grouping)	Place value counters Counters	Place value grid Written short division
6	Divide multi-digits by 2-digits (short division)	Written long division	List of multiples
6	Divide multi-digits by 2-digits (long division)	Written long division	List of multiples

Early Years

Early Years Foundation Stage

This section specifically relates to objectives set out about the 5 key counting principles (as explained above in the overview). Any objectives that linked to the four operations are listed below with the rest of the year group's objectives. Rather than having a key vocabulary section here, there are key questions listed in the 'Abstract' section for each objective to help children discuss their understanding.

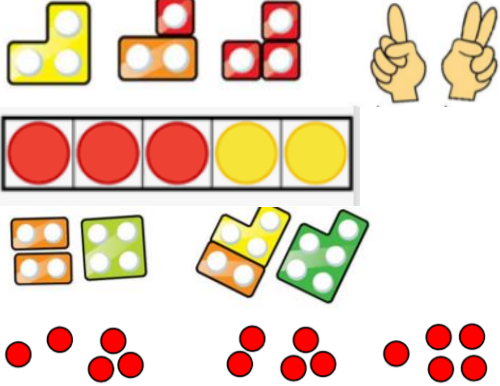
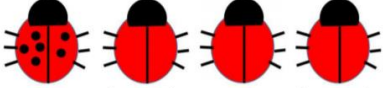
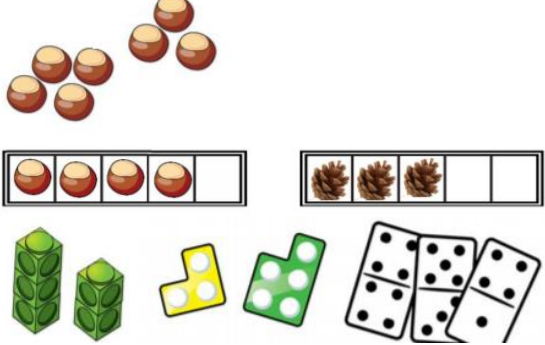

Skill	Concrete	Pictorial	Abstract
To explore different representations of numbers to 10	 	 <p>Children can draw pictures of their concrete representations such as dice, 5/10 frames, dots, draw round Numicon etc.</p>	Which representation shows ___? Which representations do not show ___? Can you
To build numbers to 20 (various representations)			make/represent a number that is/ is not ___? Can you count how many we have? Show objects on a five frame - how many do we have? How do we know without counting? What is the tallest tower you could make using 10 bricks? What's the same and what's different about the numbers we have built? Which number have I represented here? Can you find all the representations which show ___? Which do not show ___? How do you know? How many objects do we have? Can you make this object (point to one) be 6? i.e. they land on that object when they say the number 6.
To count forwards/backwards to/from 10			
To count out objects from 10			
To have an understanding of zero	Counting songs where you end up with none e.g. 5 currant buns, 5 speckled frogs, 5 little ducks, 5 little monkeys etc.		Children understand that zero represents the absence of something. How many do I need to take away to leave zero?

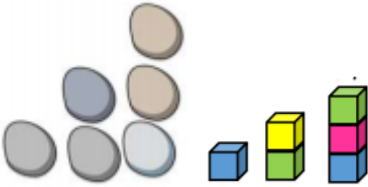
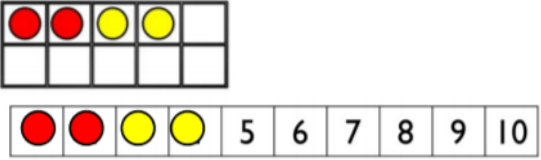

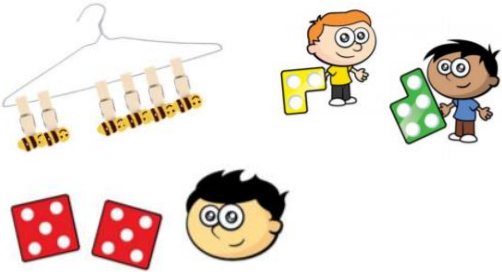
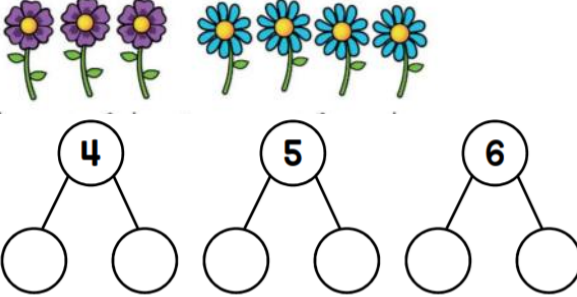
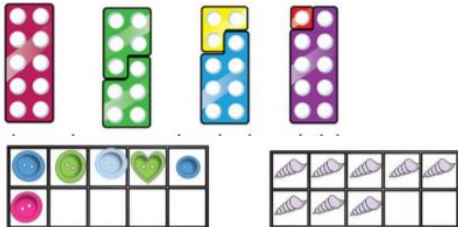
	  <p>Show me 4 claps, 2 claps, 0 claps - try with other movements.</p>		
<p>To subitise up to 5 using various representations</p>			<p>How many are there? How do you know without counting?</p>
<p>To arrange 6 - 10 in different ways to help subitise larger numbers using various representations</p>			<p>What about higher numbers? Did you see smaller groups of objects to help you know how many there were?</p>

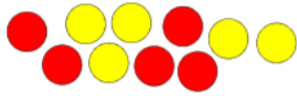
Addition

EYES

Rather than having a key vocabulary section here, there are key questions listed in the 'Abstract' section for each objective to help children discuss their understanding.

Skill	Concrete	Pictorial	Abstract
<p>To understand all quantities are made up of smaller quantities (composition)</p> <p>To explore the composition of numbers to 10</p>		<p>Children can draw pictures of their concrete representations such as 5 frames, counters, draw round Numicon, hands with different fingers showing, dominoes.</p> <p>Making 6 on ladybirds - Can you find more than one way to do it?</p> 	<p>Can you show me ___ on your fingers? Now show me a different way. How many objects can you feel in the bag? How many have I hidden (Kim's game)? Drop pebbles/coins into a bucket/tub - how many do I have? What smaller groups can you see in your number ___? Can you find all the possibilities? My domino has 6 spots - what could it look like?</p>
<p>To compare amounts</p> <p>To compare numbers to 10 (2 quantities)</p>		<p>Children can draw pictures of their concrete representations such as dice, 5 frames, dots, draw round Numicon etc.</p>  <p>Can you draw a picture to show me?</p>	<p>Which group has more/most? Which group has fewer/fewest/less/least? Do you have any equal groups? How can you check? What happens if 1 more arrives/ 1 goes away? Can you find 2 sets with the same amount? How many more to fill my 5 frame? Look at the domino - are there the same, more or fewer on the sides? Can you find a plate with more/less dots? Could this bucket have ___ pebbles? How do you know?</p>










<p>To order 3 or more quantities</p>		<p>Children can draw pictures of their concrete representations such as dice, 5 frames, dots, draw round Numicon etc.</p>	<p>How many objects does this group have? Can you find a set with more/less? Can you put these in order? What would come next? Put the dot plates in order - which one is missing?</p>
<p>To add more</p>		 <p>First there were 2 people on the bus. Then 2 more people got on the bus. Now there are 4 people on the bus. Children should be able to discuss maths stories like these using 'First, Then, Now' structure with increasing independence from a picture similar to this.</p>	<p>Show me 5 fingers, now show me 2 more - how many do we have? Did you need to count them all? Why not?</p>
<p>To recognise a quantity changes when adding more</p>			<p>How many purple flowers can you see? How many blue flowers? How many flowers altogether? Can you find 2 plates which have a total of ___ objects? Can you find more than 1 way to arrange the objects when you make the total ___?</p>
<p>To explore number bonds to 10</p>		<p>Can you draw what you found? Children can draw pictures of their concrete representations such as dice, 10 frames, dots, draw round Numicon, dominoes etc.</p>	<p>Which Numicon tile goes with mine to make 10? Roll the dice, what number did you roll? How many more do you need to make 10? Can you find a way to make 10 using 3 numbers?</p>

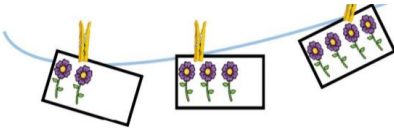


To find one more

Show me one more.

Number of the day is 3

One less	The same as	One more
		
		
		



Draw a given number of objects e.g. counters. What would 1 more be? Can you draw it?

Can you predict what will come next? Can you find 1 more than ___? Can you find a picture that is 1 more than ___ but 1 less than ___? How many are in the bag? If add 1 more, how many will there be now?

Year 1

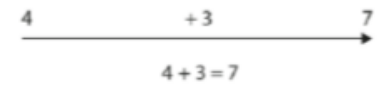
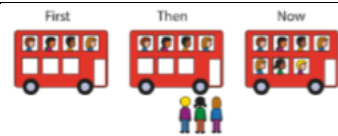
Key vocabulary

Part, whole, represent, smaller, partition, equals, addition, add, plus, more, more than, total, number sentence, add together, number bond, 1-digit number, 2-digit number, largest number, smallest number, efficient, 'first, then, now', count all, count on, maths story, make, sum, altogether, double, near double, one more, two more...ten more,

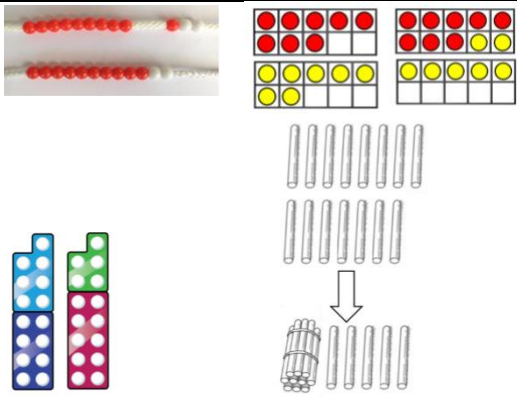
How many were there at the start? Then how many more were added? What is the total? Can the parts change place? Can the whole change place? If ___ is the whole, what could the parts be? How can we partition the whole? What number sentence would represent the parts we have partitioned the wholes in to? Do I have to start with the largest number? Why is it more efficient to start with the largest number? What could the story be? How many more to make...? How many more is... than ...? How much more is?



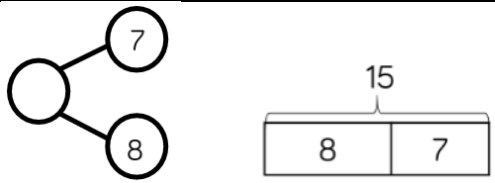
Use 'First, Then, Now' in a range of practical situations e.g. First there were 4 children on the carpet, then 3 more came. Now there are 7 children on the carpet.'



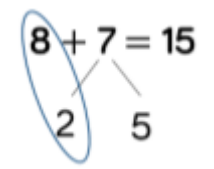
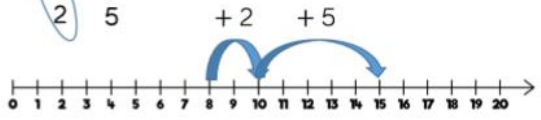
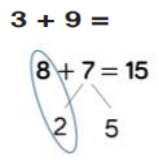
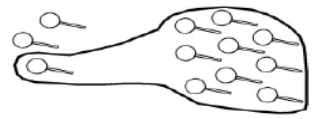
Add 1 and 2-digit numbers to 20



Highlight the importance of ten ones being equal to one ten. This is important so children understand regrouping to make 10 and making exchanges.



Regroup the picture to make 10 or partition the smaller number using the part-whole model to make 10 to make addition on the number line easier.



$$8 + 7 = 15$$

Knows number bonds and related facts to and within 20.

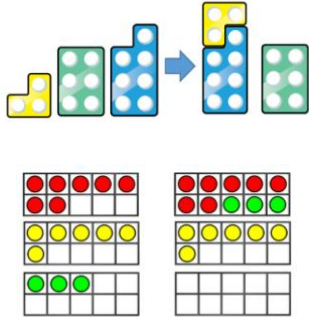
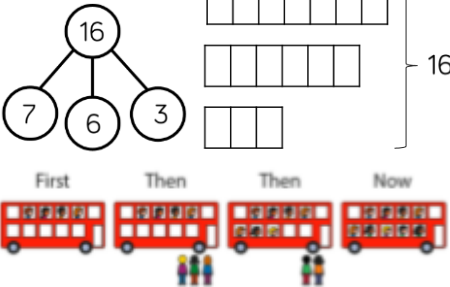
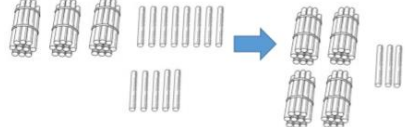
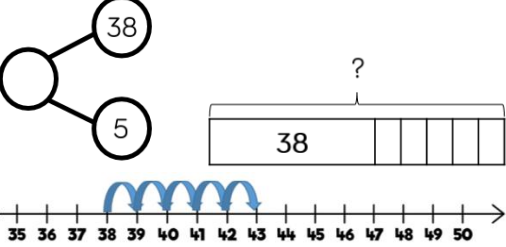
Emphasis on language in this abstract stage e.g. 1 more than 5 is equal to 6. 2 more than 5 is 7. 8 is 3 more than 5.

Year 2

Key vocabulary

Part, whole, calculation, fact family, calculate, total, altogether equals, exchange, tens, ones, related facts, number facts, multiple, column, change, exchange, mental strategy, efficient method, Addition, add, more, and, make, sum, total, altogether, Double, near double, One more, two more...ten more, one hundred more.... Equals, is the same as

If I move the objects, does the answer change? What does each circle represent on the part-whole model? How Many different number sentences are there in the fact family? What does the word multiple mean? What does the 6 represent in 63? What does the 3 represent? Will you start with 32 or 55? Why? Which column changes? Which column stays the same? Do I need to make an exchange? How many ones are the same as 1 ten? How many ones do we have? How many tens do we have? How many more to make...? How many more is.... than ...? How much more is?

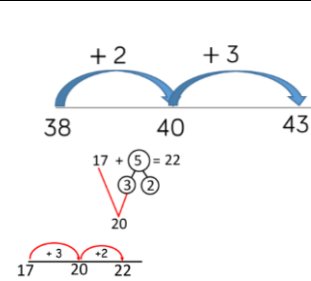
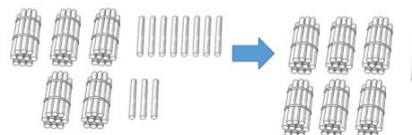
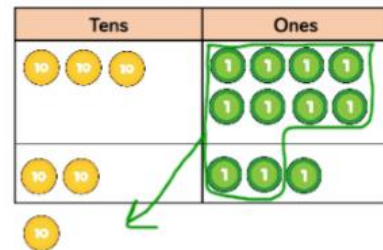
Skill	Concrete	Pictorial	Abstract
<p>Add three 1-digit numbers</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Encourage children to find bonds to 10 or doubles here for efficiency. Manipulatives that highlight bonds to 10 are especially useful.</p> </div>	 <p>The concrete representation shows three groups of blocks: a single yellow block (1), a 2x2 grid of green blocks (4), and a 2x2 grid of blue blocks (4). An arrow points to a new arrangement: a 2x2 grid of blue blocks (4), a 2x2 grid of green blocks (4), and a single yellow block (1). Below this are two ten frames. The first has 7 red dots, 2 yellow dots, and 1 green dot. The second has 2 red dots, 2 yellow dots, and 2 green dots. Below the ten frames are three hands holding different combinations of colored dots.</p>	 <p>The pictorial representation shows a number bond with 16 at the top, branching into 7, 6, and 3. To the right are three rows of ten frames: the first has 6 empty boxes, the second has 6 empty boxes, and the third has 4 empty boxes. A bracket on the right groups these as 16. Below is a bus stop diagram with four buses labeled 'First', 'Then', 'Then', and 'Now'. The first bus has 4 people, the second has 7, the third has 6, and the fourth has 17. Below the bus stop is a box with the text: 'Build on and extend Year 1 knowledge to include 'First, Then, Then, Now'.'</p>	<p style="text-align: center;">$7 + 6 + 3 = 16$</p> <p style="text-align: center;">10</p> <div style="border: 1px solid black; padding: 5px; margin: 5px auto; width: 80%;"> <p style="text-align: center;">$7 + 6 + 3 = 16$</p> </div> <p style="text-align: center;">$4 + 7 + 6 = 10 + 7$</p> <p style="text-align: center;">10</p> <p style="text-align: center;">$= 17$</p> <p style="text-align: center;">Combine the 2 numbers that make 10 then add on the third.</p>
<p>Add 1 and 2-digit numbers to 100</p>	 <p>The concrete representation shows three bundles of ten (tens) and eight individual ones (ones). An arrow points to four bundles of ten and three individual ones.</p>	 <p>The pictorial representation shows a number bond with 38 at the top, branching into an empty circle and 5. To the right is a ten frame with 38 in the first two boxes and 5 empty boxes. Below is a number line from 35 to 50. Blue arrows show a jump from 38 to 43.</p>	<p style="text-align: center;">$38 + 5 = 43$</p> <div style="border: 1px solid black; padding: 5px; margin: 5px auto; width: 80%;"> <p>Children should use knowledge of number bonds for efficiency e.g. $8 + 5 = 13$ so $38 + 5 = 43$.</p> </div> <p style="text-align: center;">Related facts should be explored here:</p>

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Straws and hundred squares are particularly useful for helping children find the bond to 10.

Add two 2-digit numbers
- No exchange first,
then with exchange

At this point, formal written method should be used alongside concrete (base 10, straws or place value counters). Straws become less efficient as numbers increase. Only use place value counters when children have a clear understanding that 10 ones is equal to 1 ten using Base 10.



Use part-whole model and number line to model.

$17 + 5 = 22$

22	
17	5

Explore related facts

$17 + 5 = 22$

$22 = 17 + 5$

$5 + 17 = 22$

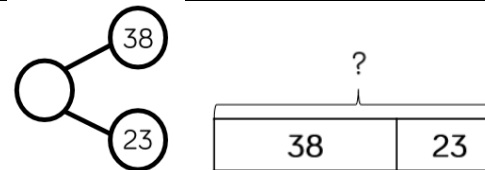
$22 = 5 + 17$

$22 - 17 = 5$

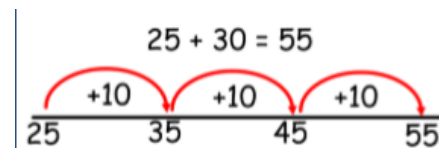
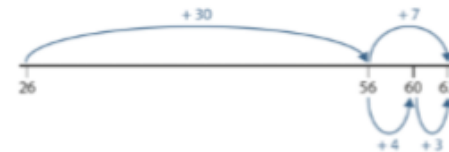
$17 = 22 - 5$

$22 - 5 = 17$

$5 = 22 - 17$



$26 + 37 = 63$
 $26 + 30 + 7$



$38 + 23 = 61$

Expanded method (if necessary)

$$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$$

$27 + 10 = 37$

$27 + 20 = 47$

$27 + \square = 57$

$\square + 30 = 67$

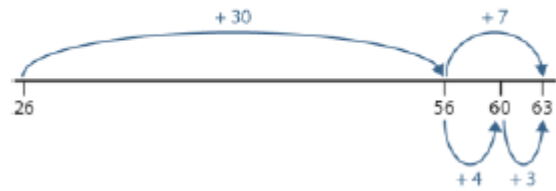
38

$23 +$

$\begin{array}{r} 61 \\ 1 \end{array}$

Use column addition script, see Appendix 2

$$26 + 30 + 7$$



Children can use a blank number line to count on and find a total. Encourage them to jump to multiples of 10 for efficiency.

Year 3

Key vocabulary

Sum, altogether, total, addition, add, more, and, make, sum, total, altogether, double, near double, one more, two more.... ten more, one hundred more.....

What is __ hundred and __ hundreds equal to? Which column do I need to focus on? How many ones do we have? How many tens do we have? Do we need to make an exchange? How many tens is equal to one hundred? How many more to make...? How many more is.... than ...? How much more is?

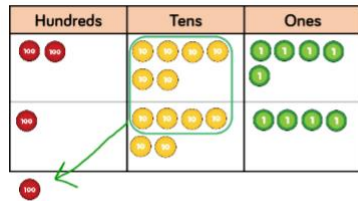
Skill

Concrete

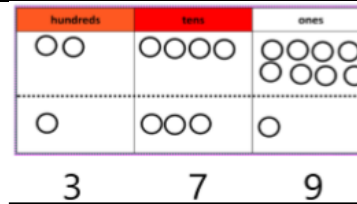
Pictorial

Abstract

Add with up to 3-digits



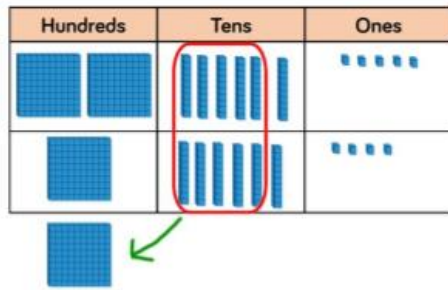
$$\begin{array}{r} 265 \\ 164 + \\ \hline 429 \\ 1 \end{array}$$



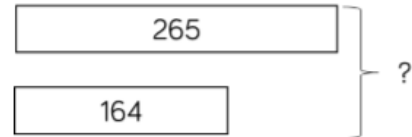
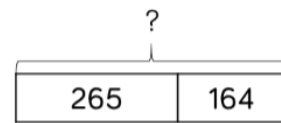
Children then draw counters or Base 10 on a place value chart.

$$265 + 164 = 429$$

248	265
131 +	164 +
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>
379	429
	<hr style="width: 100%;"/>
	1



Base 10 and place value counters are most effective for numbers with 3 digits. Children should write out calculation alongside concrete to help them make links to column method later on. Plain counters can also be used on a place value grid. Calculations with no regrouping required should always be taught before those that have regrouping i.e. need to make an exchange.



Children can use a blank number line to count on and find a total. Encourage them to jump to multiples of 10 for efficiency.

Use column addition script, see Appendix 2

Year 4

Key vocabulary

Sum, altogether, total, addition, add, more, and, make, sum, total, altogether, double, near double, one more, two more.... ten more, one hundred more..... Equals, is the same as

How many more to make...? How many more is.... than ...? How much more is? Where would these digits go on a place value chart? Why do we make both numbers when we add? Why is it important to put the digits in the correct column? Is it more difficult to add 3-digit or 4-digit numbers without exchanging? Why? How many ones do we need to exchange for one ten? Why is it so important to show the exchanged digit on the column method? Which columns are affected if there are more than ten tens? Why is it important to line up the digits in the correct column when adding numbers with different amounts of digits? Why do you have to add the digits right to left, starting with the smallest place value column?

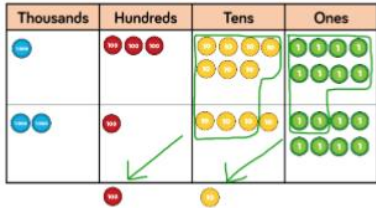
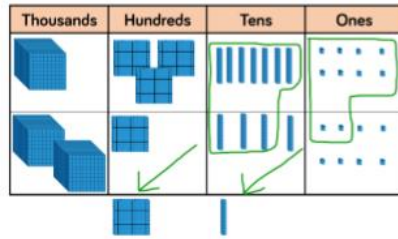
Skill

Concrete

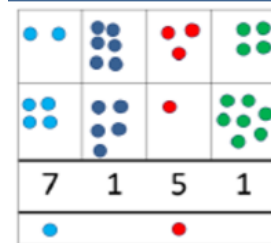
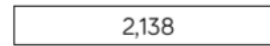
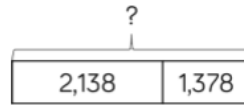
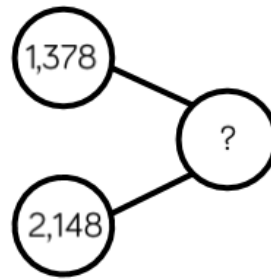
Pictorial

Abstract

Add with up to 4-digits

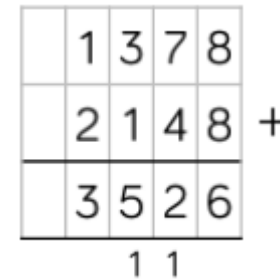


Base 10 and place value counters are most effective for numbers with 3 digits. Children should write out calculation alongside concrete to help them make links to column method later on. Plain counters can also be used on a place value grid.



Children can then draw counters on a place value chart.

$$1,378 + 2,148 = 3,526$$



Use column addition script, see Appendix 2

Year 5 and 6

Key vocabulary

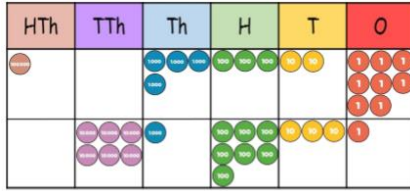
Addition, add, more, and, make, sum, total, altogether, Double, near double, half, halve, One more, two more...ten more, one hundred more....., Equals, is the same as

How many more to make...? How many more is.... than ...? How much more is? Can we make an exchange? Are there any ones remaining? Which columns are affected if there are more than ten tens altogether? Why is it important to line up the digits in the correct column when adding numbers with different amount of digits? Will you have to exchange? How do you know which columns will be affected? Does it matter that the two numbers don't have the same amount of digits? Which number goes on top in the calculation? Does it affect the answer?

Skill

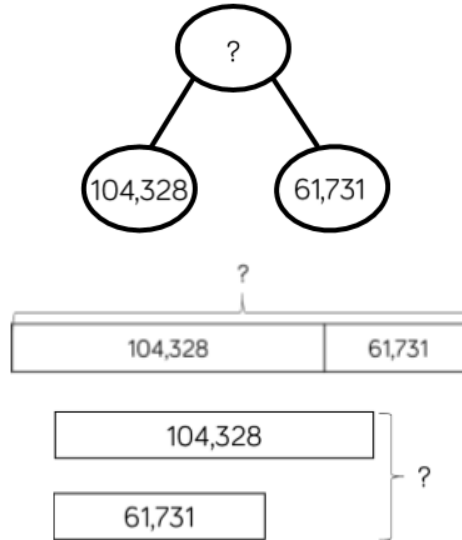
Add with more than 4-digits

Concrete



Place value counters and plain counters on place value grids are most effective when adding numbers with more than 4 digits.

Pictorial



Abstract

$$\begin{array}{r} 89,472 \\ 63,673 \\ \hline 3,016 + \\ \hline 156,161 \\ \hline 1\ 1\ 1\ 1 \end{array}$$

1	0	4	3	2	8
	6	1	7	3	1
1	6	6	0	5	9

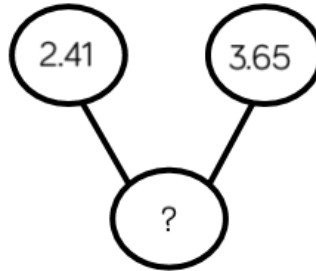
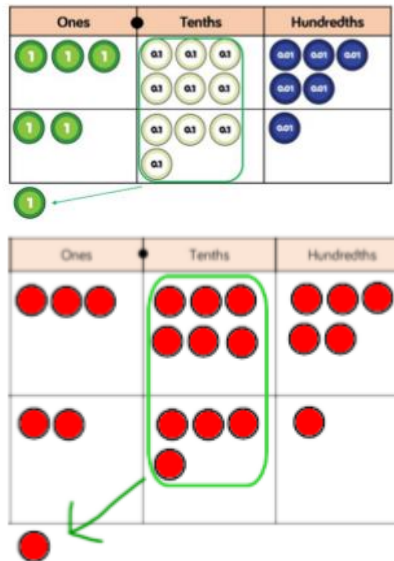
1

104,328 + 61,731 = 166,059

At this stage children should be encouraged to work in the abstract, using column method to add more efficiently.

Use column addition script, see Appendix 2

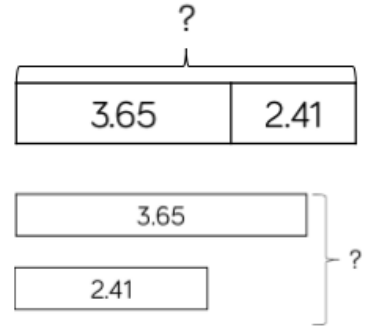
Add with up to 3 decimal places



$$\begin{array}{r} \pounds 127.67 \\ \pounds 38.45 + \\ \hline \pounds 166.12 \\ \hline 1\ 1\ 1 \end{array}$$

3.65 + 2.41 = 6.06

Place value counters and plain counters on place value grids are most effective when adding numbers with 1, 2 and then 3 decimal places.



$$\begin{array}{r} 1.437 \\ 0.600 \\ 3.020 \\ \hline 4.057 \\ \hline 1 \end{array} \quad \begin{array}{r} 3.65 \\ 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$$

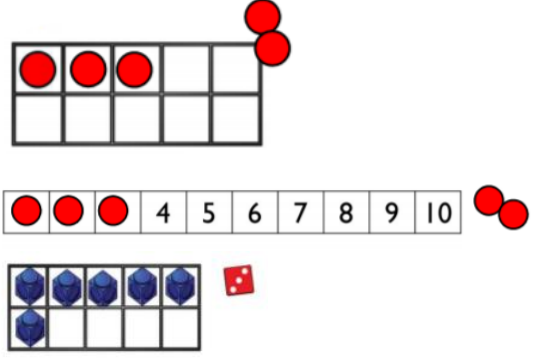

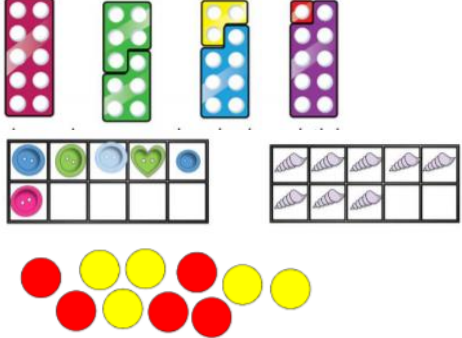
Include zeros for place holders.

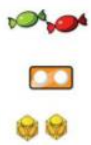


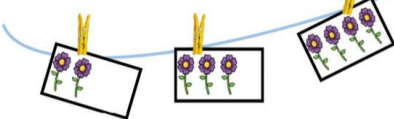
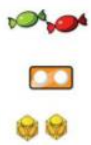


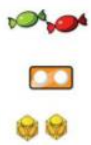


Use column addition script, see Appendix 2

Subtraction

EYES

Rather than having a key vocabulary section here, there are key questions listed in the 'Abstract' section for each objective to help children discuss their understanding.

Skill	Concrete	Pictorial	Abstract
<p>To take away</p> <p>To recognise a quantity changes when taking away</p>		 <p>First there were 5 people on the bus. Then 2 people got off the bus. Now there are 3 people on the bus. Children should be able to discuss maths stories like these using 'First, Then, Now' structure with increasing independence from a picture similar to this.</p>	<p>If I have ___ and I take away ___, how many are left? Do we always need to count how many are left? Why not? How many cubes did we start with? How many have we got now? How many cubes did I take away? How did you work it out? Do we have more/fewer cubes now?</p>
<p>To explore subtraction facts to 10</p>		<p>Can you draw what you found? Children can draw pictures of their concrete representations such as dice, 10 frames, dots, draw round Numicon, dominoes etc.</p>	<p>Which Numicon tile goes with mine to make 10? Roll the dice, what number did you roll? How many do you need to take away from 10 to make ___?</p>

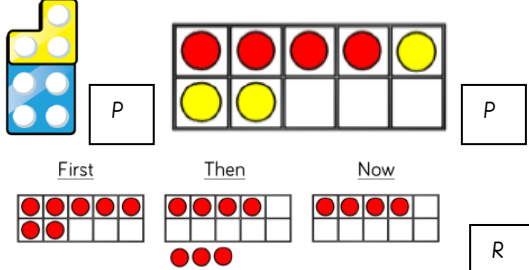
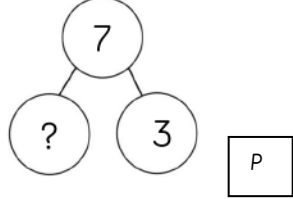
<p>To find one less</p>	<p>Show me one less/fewer.</p> <p>Number of the day is 3</p> <table border="1" data-bbox="414 175 772 391"> <tr> <td data-bbox="414 175 533 215">One less</td> <td data-bbox="533 175 660 215">The same as</td> <td data-bbox="660 175 772 215">One more</td> </tr> <tr> <td data-bbox="414 215 533 391">  </td> <td data-bbox="533 215 660 391">  </td> <td data-bbox="660 215 772 391">  </td> </tr> </table> 	One less	The same as	One more				<p>Draw a given number of objects e.g. counters. What would 1 less/fewer be? Can you draw it?</p>	<p>Can you predict what will come next? Can you find 1 less than ___? Can you find a picture that is 1 less than ___ but 1 more than ___? How many are in the bag? If take 1 away, how many will there be now?</p>
One less	The same as	One more							
									

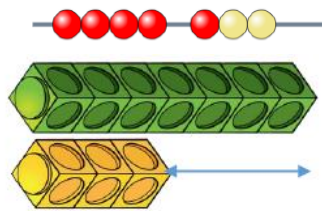
Year 1

Key vocabulary

Count all, count on, subtract, subtraction, take away, 'first, then, now', calculations, count back, difference, find the difference, Take away, subtract, One less, two less, ten less.. difference between, equals, is the same as, number bonds/pairs, missing number

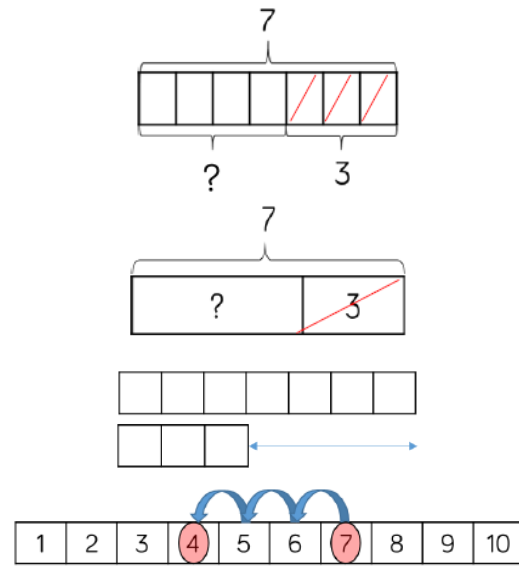
How many objects were there to start with? How many left? What number can we use to show that nothing has been taken away? What is the whole? What are the parts? If ___ is the whole and ___ is one part, what is the other part? What is the same/different about these calculations? What number comes before___? What does difference mean? What strategy can we use to help us find the difference? How many are left/left over? How many have gone? How many fewer is..... than....? How much less is..?

Skill	Concrete	Pictorial	Abstract
<p>Subtract two 1-digit numbers to 10</p>	 <p>First: 7 red counters in a ten frame. Then: 3 red counters are crossed out. Now: 4 red counters remain in the ten frame, with 3 red counters shown below.</p>	 <p>A number bond with 7 in the top circle, a question mark in the bottom-left circle, and 3 in the bottom-right circle. A box labeled 'P' is next to it.</p>	<div style="border: 1px solid black; border-radius: 15px; padding: 10px; text-align: center;"> $7 - 3 = 4$ </div>



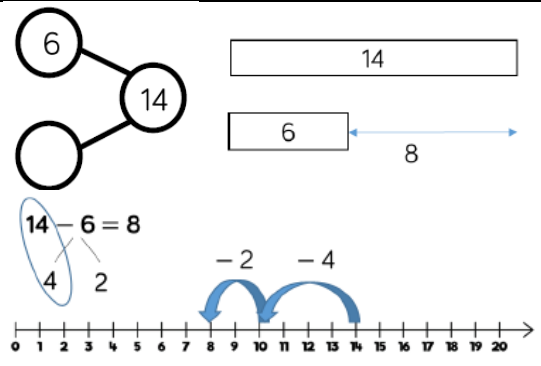
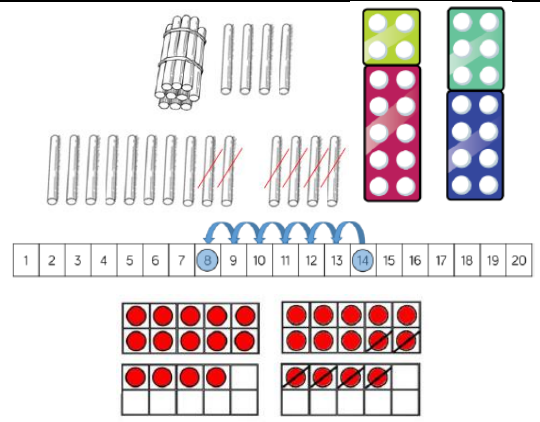
R

D

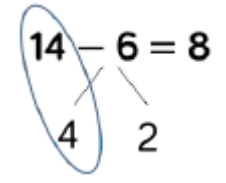


Subtraction can take several forms: partitioning (P), reduction (R) and find the difference (D). It is important the model and representations shown best support the children in the type of subtraction they will be doing. Next to each model, the initials show which model represents each type of question best.

Subtract 1 and 2-digit numbers to 20



$$14 - 6 = 8$$

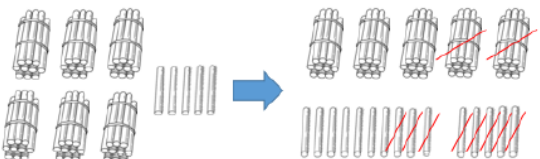
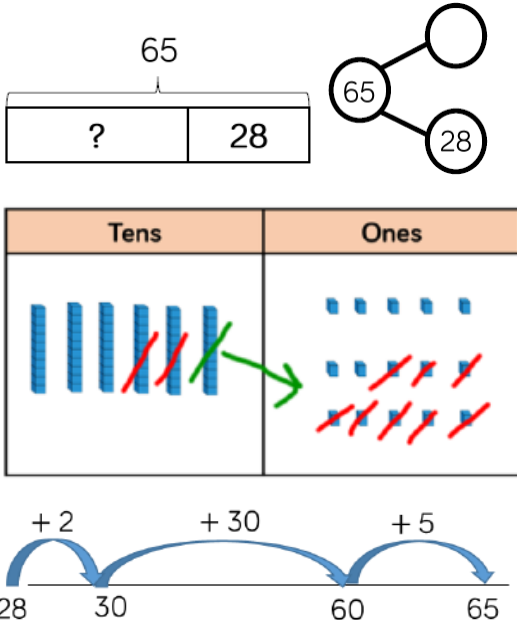


Year 2

Key vocabulary

Subtract, take away, count back, backwards, subtraction, efficient method, mental strategy, difference, find the difference between, half, Subtract,, take away, half, halve, Equals, is the same as, One less, two less, ten less, one hundred less, Difference between

Are we counting forwards or backwards on a number line? How can we show the takeaway? Have we got enough ones to subtract? Can you partition the number into tens and ones? How could you represent the problem? What is the value of the digits? Which number do we need to make, the larger or the smaller number? How many have we got left? What's the difference between the numbers? DO we need to subtract the ones first? Why? How many are left/left over? How many have gone? How many fewer is..... than....? How much less is...?

Skill	Concrete	Pictorial	Abstract
<p>Subtract 1 and 2-digit numbers to 100</p> <p>and</p> <p>Subtract two 2-digit numbers</p>	<div style="text-align: center;">  </div> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Straws become less efficient as numbers become larger. Children should be encouraged to use formal column method should be written alongside concrete calculations.</p> <p>Also children draw a blank number line to find the difference, counting to multiples of 10 for efficiency.</p> </div>	<div style="text-align: center;">  </div>	<div style="text-align: center; border: 1px solid black; padding: 10px; width: fit-content; margin: 0 auto;"> $65 - 28 = 37$ </div> <div style="text-align: center; margin-top: 20px;"> $\begin{array}{r} \overset{5}{\cancel{6}}\overset{1}{5} \\ - 28 \\ \hline 37 \end{array}$ </div> <p style="text-align: center;"><i>Use column subtraction script, see Appendix 2</i></p>

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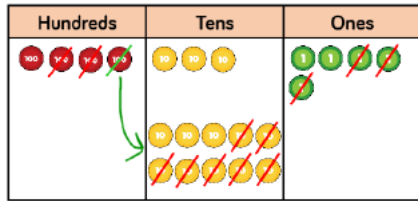
Year 3

Key vocabulary

half, halve, subtract, take away, One less, two less, ten less, one hundred less, Difference between, Equals, is the same as, Number patterns

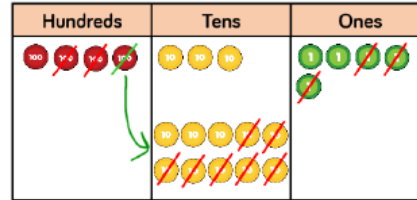
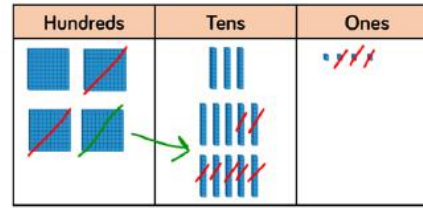
Why do all these subtractions require an exchange? How many tens can we add to ___ without needing to make an exchange? How many tens do we have? How many tens do we need to make an exchange for a hundred? Do we need to write a 0 in the hundreds column when there are no hundreds? What is the sum of the numbers? How many have we got altogether? How many are left/left over? How many have gone? How many fewer is..... than....? How much less is..?

Skill	Concrete	Pictorial	Abstract
Subtract with up to 3-digits			<div style="border: 1px solid black; border-radius: 10px; padding: 5px; display: inline-block;"> $435 - 273 = 262$ </div> $ \begin{array}{r} \overset{3}{4} \overset{1}{3} 5 \\ - 273 \\ \hline 262 \end{array} $ <p><i>Use column subtraction script, see Appendix 2</i></p>



Base 10 and Place value counters are the most efficient concrete resource here. Plain counters on a PV grid can also be useful.

Ensure column method is written out alongside concrete and pictorial calculations so the link can be made with the abstract.



Year 4

Key vocabulary

Subtract, take away, One less, two less, ten less, one hundred less, Difference between, Equals, is the same as

How many are left/left over? How many have gone? How many fewer is..... than....? How much less is...? Does it matter which number is at the top of the subtraction? Do you need to make both numbers when you subtract? Why? What happens when you can't subtract 9 ones from 7 ones? What would we need to do? How would you teach someone else to use column subtraction with exchange? How do we indicate the exchange on the written method? When do we need to exchange within a column subtraction? What happens if there's a zero in the next column? How do we exchange?

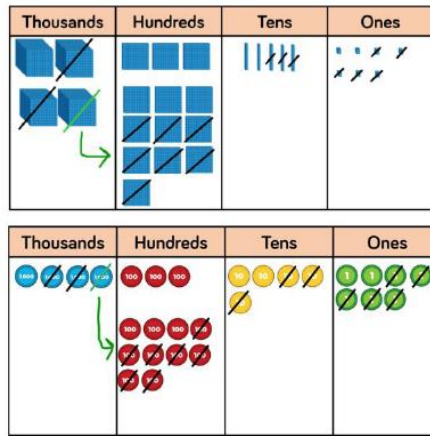
Skill

Concrete

Pictorial

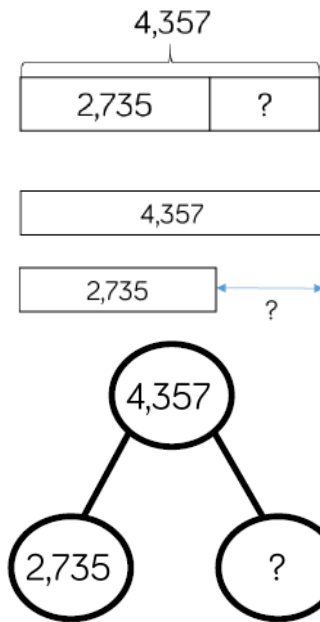
Abstract

Subtract with up to 4-digits



Base 10 and Place value counters are the most efficient concrete resource here. Plain counters on a PV grid can also be useful.

Ensure column method is written out alongside concrete and pictorial calculations so the link can be made with the abstract.



$$\begin{array}{r}
 3 \ 1 \\
 \cancel{4}357 \\
 - 2735 \\
 \hline
 1622
 \end{array}$$

$$4,357 - 2,735 = 1,622$$

Use column subtraction script, see Appendix 2

Year 5/6

Key vocabulary Subtract, take away, One less, two less, ten less, one hundred less, Difference between, Equals, is the same as
 How many are left/left over? How many have gone? , How many fewer is..... than....?, How much less is..? When do we need to exchange in subtraction? How do we indicate the exchange on the written method? What happens if there is a zero in the next column? How do we exchange? Does it matter which number goes on top? Why?

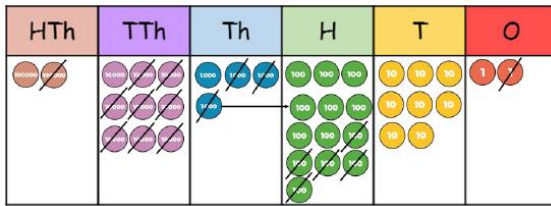
Skill

Concrete

Pictorial

Abstract

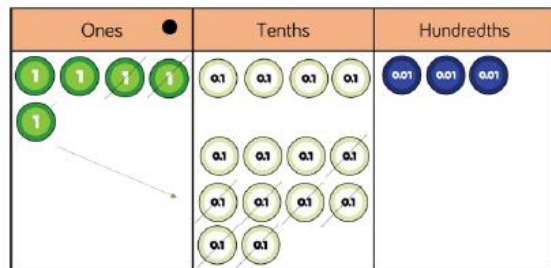
Subtract with more than 5-digits



Place value or plain counters are the most efficient concrete resource here.

At this stage, children should be encouraged to work in the abstract, using column method to subtract larger numbers more efficiently.

Subtract with up to 3 decimal places



Place value or plain counters are the most efficient concrete resource here.

Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this into context when subtracting money and other measures.

294,382

182,501

?

294,382

182,501

?

294,382

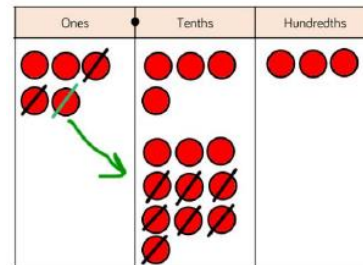
182,501

?

2.7

?

5.43



2	9	3	13	8	2
1	8	2	5	0	1
1	1	1	8	8	1

$$294,382 - 182,501 = 111,881$$

Use column subtraction script, see Appendix 2

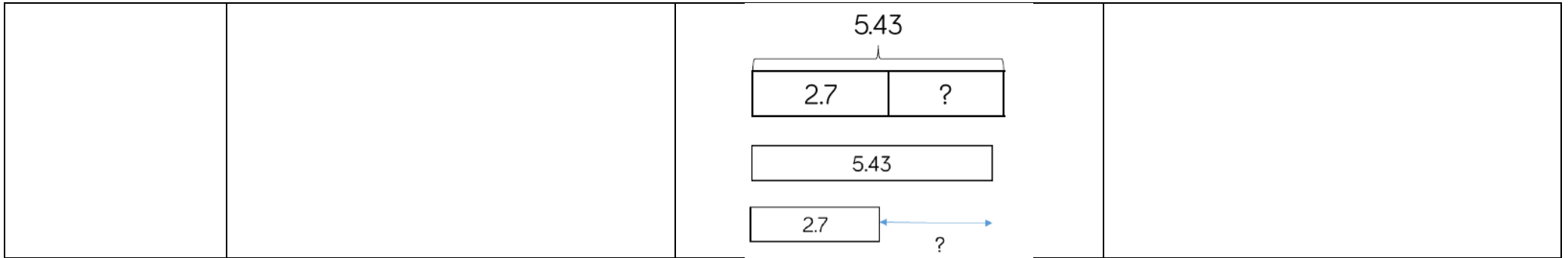
4 1
5.43

2.7 -

2.73

$$5.43 - 2.7 = 2.73$$

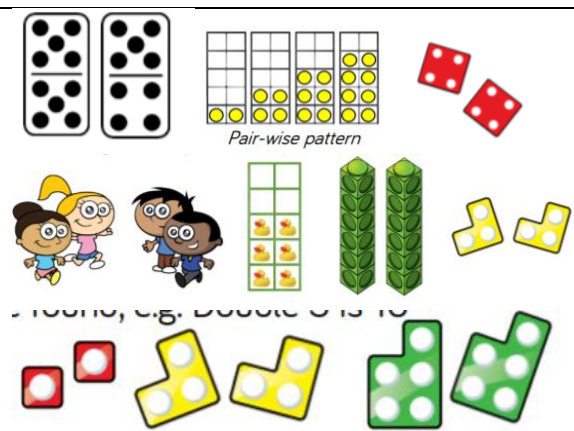

Use column subtraction script, see Appendix 2



Multiplication

EYES

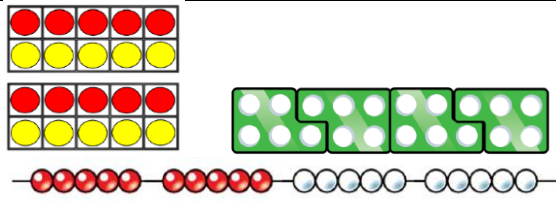
Rather than having a key vocabulary section here, there are key questions listed in the 'Abstract' section for each objective to help children discuss their understanding.

Skill	Concrete	Pictorial	Abstract
<p>To double numbers to 10</p> <p>To know that double means 'twice as many'</p>	 <p style="text-align: center; font-size: small;">Pair-wise pattern</p>	 <p>Children to draw doubles from their concrete representations pictorial e.g. draw round Numicon, draw dominoes, cubes, counters on ten frames etc.</p>	<p>Which of these shows a double? Which does not show a double? How do you know? I've made 2, can you make double that amount? Double ___ is ___.</p>

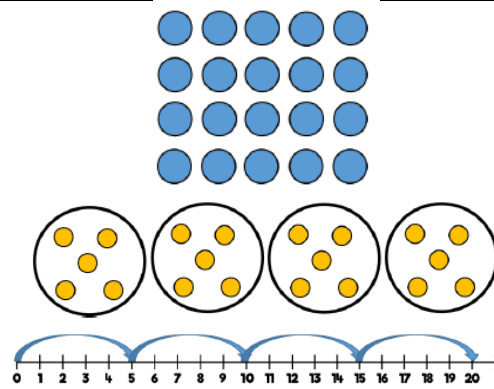
Year 1/2

Key vocabulary	<p>Pairs, groups of __, step counting, step count, count in __s, there are __ in each group, there are __ groups, there are __ altogether, total, equal groups, describe the groups, equal, unequal, arrays, row, column, doubles, not doubles, repeated addition, total, multiply, multiplication, lots of, times, times table, grouping, doubling, multiplication, multiply, multiplied by, multiple, groups of, times, once, twice, three times ... ten times, repeated addition, Multiplication table, multiplication fact, Doubling</p> <p>Will __ appear on our number line or not? Are the groups equal or unequal? How can we make them equal? How many equal groups can you see? How can we represent this problem? How many groups? How many in each group? How many altogether? How can I record my array with a number sentence? How do you know this has been doubled? If double 2 is 4 what is double 20? What does the __ represent? How many __ goes into?</p>		
Skill	Concrete	Pictorial	Abstract

Solve one-step problems with multiplication



As infants, multiplication is shown as repeated addition in various ways. Year 2 is when the multiplication symbol is introduced. There is no expectation that Year 1 record multiplication formally i.e. a number sentence. They will work more concrete and pictorially to show



$$5 + 5 + 5 + 5 = 20$$

$$4 \times 5 = 20$$

$$5 \times 4 = 20$$

One bag holds 5 apples.
How many apples do 4 bags hold?

Year 3/4

Key vocabulary

Multiplication, multiply, multiplied by, multiple, factor, groups of, times, product, once, twice, three times ... ten times, repeated addition, equal groups of Multiplication table, multiplication fact, division fact, doubling, halving, array, row, column, factor, times, product, remainder, squared, cubed,

How can we represent the groups? What does lots of mean? How many ___s go into ___? What other times table will help us answer this question? What other facts can you link to this one?

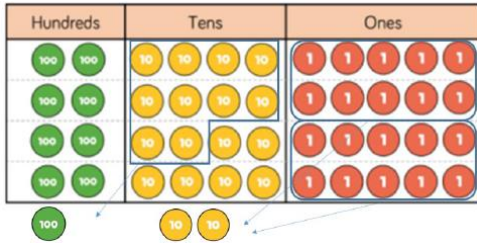
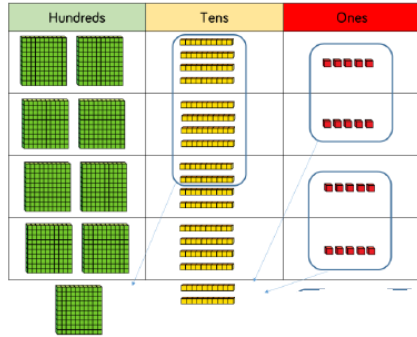
Skill	Concrete	Pictorial	Abstract																																																																	
<p>Multiply 2-digit by 1-digit numbers</p>			<table border="1" style="margin-bottom: 10px; border-collapse: collapse; text-align: center;"> <tr><td></td><td>H</td><td>T</td><td>O</td><td></td></tr> <tr><td></td><td></td><td>3</td><td>4</td><td></td></tr> <tr><td></td><td></td><td></td><td>5</td><td>×</td></tr> <tr><td colspan="4" style="border-top: 1px solid black;"></td><td></td></tr> <tr><td></td><td></td><td>2</td><td>0</td><td>(5 × 4)</td></tr> <tr><td>+</td><td>1</td><td>5</td><td>0</td><td>(5 × 30)</td></tr> <tr><td colspan="4" style="border-top: 1px solid black;"></td><td></td></tr> <tr><td></td><td>1</td><td>7</td><td>0</td><td></td></tr> </table> <table border="1" style="margin-bottom: 10px; border-collapse: collapse; text-align: center;"> <tr><td></td><td>H</td><td>T</td><td>O</td><td></td></tr> <tr><td></td><td></td><td>3</td><td>4</td><td></td></tr> <tr><td></td><td></td><td></td><td>5</td><td>×</td></tr> <tr><td colspan="4" style="border-top: 1px solid black;"></td><td></td></tr> <tr><td></td><td>1</td><td>7</td><td>0</td><td></td></tr> </table> <p style="text-align: center; margin-bottom: 0;">1 2</p> <p style="font-size: 24px; font-weight: bold; margin: 0;">$34 \times 5 = 170$</p>		H	T	O				3	4					5	×								2	0	(5 × 4)	+	1	5	0	(5 × 30)							1	7	0			H	T	O				3	4					5	×							1	7	0	
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	<p>The place value counters should be used to support the understanding of the method rather than the multiplication itself. Times table knowledge should be used for this.</p>																																																																			

			<p>24 × 6 becomes</p> $ \begin{array}{r} 24 \\ \times 6 \\ \hline 144 \\ \hline \end{array} $ <p>Answer: 144</p>
--	--	--	--

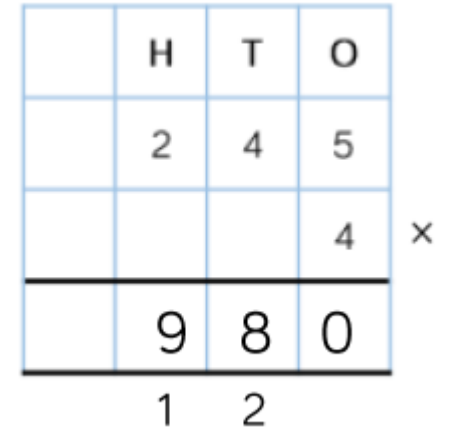
Year 4

Key vocabulary	<p>Multiplication, multiply, multiplied by, multiple, factor, groups of, times, product, once, twice, three times ... ten times, repeated addition, Multiplication table, multiplication fact, Doubling, Array, row, column, equal groups of, Inverse, square, squared cube, cubed</p> <p>Can you calculate the multiplication mentally or do you need to write down your method? How does multiplication link to addition? How does partitioning help you to multiply the numbers? Which column shall we start with, the tens or the ones? How is multiplying a 3-digit by 1-digit number similar to multiplying a 2-digit by 1-digit number?</p>		
	Skill	Concrete	Pictorial

Multiply 3-digit
by 1-digit
numbers



Children should now be moving towards using short, formal written method. Base 10 and place value counters can continue to be used where there is a limited number of exchanges to help underpin understanding of this method. As numbers become larger, move away from using resources.



$$245 \times 4 = 980$$

342 × 7 becomes



Answer: 2394

Year 5/6

Key vocabulary

Multiplication, multiply, multiplied by, multiple, factor, common factor, groups of, times, product, once, twice, three times ... ten times, repeated addition, equal groups of, Multiplication table, multiplication fact, array, row, column, Doubling, Inverse, square, squared cube, cubed, prime number, composite number,

Can you calculate the multiplication mentally or do you need to write down your method? How does multiplication link to addition? How does partitioning help you to multiply the numbers? Which column shall we start with, the tens or the ones? How is multiplying a 3-digit by 1-digit number similar to multiplying a 2-digit by 1-digit number? Which direction do the digits move if you multiply by 10, 100 or 1000? How many places do you move to the left? When we have an empty place value column to the right of our digits, what number do we use as a place holder? Can you explain multiplying $__$ by $__$? Explain how you did this.

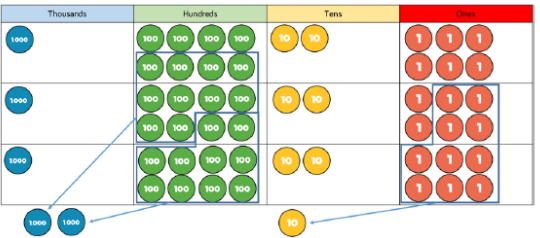
Skill	Concrete	Pictorial	Abstract																																																						
<p>Multiply 2-digit by 4-digit numbers</p>	<div style="border: 1px solid black; padding: 10px; width: fit-content; margin: auto;"> <p>Children should now be confident in the written method. If they are still struggling with times tables, provide multiplication grids to support when they are focusing on the use of the method. Consider where exchanged digits are placed and make sure this is consistent.</p> </div>		<table border="1" style="margin: auto; border-collapse: collapse;"> <tr> <td style="padding: 5px;">TTh</td> <td style="padding: 5px;">Th</td> <td style="padding: 5px;">H</td> <td style="padding: 5px;">T</td> <td style="padding: 5px;">O</td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px;"></td> <td style="padding: 5px; text-align: center;">2</td> <td style="padding: 5px; text-align: center;">7</td> <td style="padding: 5px; text-align: center;">3</td> <td style="padding: 5px; text-align: center;">9</td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px; text-align: center;">2</td> <td style="padding: 5px; text-align: center;">8</td> <td style="padding: 5px; text-align: center;">×</td> </tr> <tr> <td style="padding: 5px; text-align: center;">2</td> <td style="padding: 5px; text-align: center;">1</td> <td style="padding: 5px; text-align: center;">9</td> <td style="padding: 5px; text-align: center;">1</td> <td style="padding: 5px; text-align: center;">2</td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px; text-align: center;">2</td> <td style="padding: 5px; text-align: center;">5</td> <td style="padding: 5px; text-align: center;">3</td> <td style="padding: 5px; text-align: center;">7</td> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px; text-align: center;">5</td> <td style="padding: 5px; text-align: center;">4</td> <td style="padding: 5px; text-align: center;">7</td> <td style="padding: 5px; text-align: center;">8</td> <td style="padding: 5px; text-align: center;">0</td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px; text-align: center;">1</td> <td style="padding: 5px;"></td> <td style="padding: 5px; text-align: center;">1</td> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px; text-align: center;">7</td> <td style="padding: 5px; text-align: center;">6</td> <td style="padding: 5px; text-align: center;">6</td> <td style="padding: 5px; text-align: center;">9</td> <td style="padding: 5px; text-align: center;">2</td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px; text-align: center;">1</td> </tr> </table> <p style="text-align: center; font-weight: bold; margin-top: 10px;">$2,739 \times 28 = 76,692$</p>	TTh	Th	H	T	O			2	7	3	9					2	8	×	2	1	9	1	2		2	5	3	7			5	4	7	8	0		1		1				7	6	6	9	2							1
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Year 5

Key vocabulary

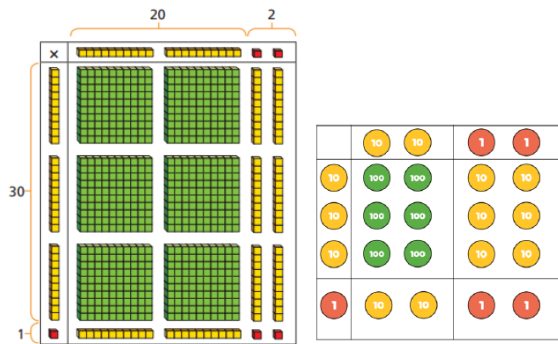
Multiplication, multiply, multiplied by, multiple, factor, common factor, groups of, times, product, once, twice, three times ... ten times, repeated addition, equal groups of, Multiplication table, multiplication fact, array, row, column, Doubling, Inverse, square, squared cube, cubed, prime number, composite number,

What is important to remember as we begin multiplying by the tens number? Can you calculate the multiplication mentally or do you need to write down your method? How does multiplication link to addition? How does partitioning help you to multiply the numbers? Which column shall we start with, the tens or the ones? How is multiplying a 3-digit by 1-digit number similar to multiplying a 2-digit by 1-digit number? Which direction do the digits move if you multiply by 10, 100 or 1000? How many places do you move to the left? When we have an empty place value column to the right of our digits, what number do we use as a place holder? Can you explain multiplying ___ by ___? Explain how you did this.

Skill	Concrete	Pictorial	Abstract																														
<p>Multiply 4-digit by 1-digit numbers</p>	 <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Place value counters are the best manipulative to support here. If they are still struggling with times tables, provide multiplication grids to support so they can focus on the use of the written method.</p> </div>		<p>2741 × 6 becomes</p> $ \begin{array}{r} 2741 \\ \times 6 \\ \hline 16446 \\ \hline 42 \end{array} $ <p>Answer: 16 446</p> <div style="border: 1px solid black; border-radius: 10px; padding: 5px; display: inline-block; margin: 10px 0;"> $1,826 \times 3 = 5,478$ </div> <table border="1" style="margin-top: 10px; border-collapse: collapse; text-align: center;"> <tr> <td></td> <td>Th</td> <td>H</td> <td>T</td> <td>O</td> <td></td> </tr> <tr> <td></td> <td>1</td> <td>8</td> <td>2</td> <td>6</td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td>3</td> <td>×</td> </tr> <tr> <td></td> <td>5</td> <td>4</td> <td>7</td> <td>8</td> <td></td> </tr> <tr> <td></td> <td>2</td> <td></td> <td>1</td> <td></td> <td></td> </tr> </table>		Th	H	T	O			1	8	2	6						3	×		5	4	7	8			2		1		
	Th	H	T	O																													
	1	8	2	6																													
				3	×																												
	5	4	7	8																													
	2		1																														

Multiply 2-digit by 2-digit numbers

Use the area model to help children understand the size of the numbers they are using. This links to finding the area of a rectangle by finding the space covered by Base 10. The grid method matches the initial written method before moving on to the formal written multiplication method.



×	20	2
30	600	60
1	20	2

$$22 \times 31 = 682$$

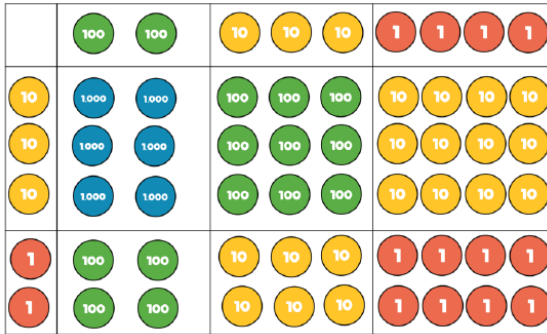
	H	T	O	
		2	2	
		3	1	×
<hr/>				
		2	2	
	6	6	0	
<hr/>				
	6	8	2	

24 × 16 becomes

$$\begin{array}{r}
 24 \\
 16 \times \\
 \hline
 240 \\
 144 \\
 \hline
 384
 \end{array}$$

Answer: 384

Multiply 2-digit
by 3-digit
numbers



x	200	30	4
30	6,000	900	120
2	400	60	8

Th	H	T	O
	2	3	4
		3	2
<hr/>			
	4	6	8
¹ 7	¹ 0	2	0
<hr/>			
7	4	8	8

$$234 \times 32 = 7,488$$

124 x 26 becomes

		1	2	
		1	2	4
			2	6 ×
<hr/>				
2	4	8	0	
	7	4	4	
<hr/>				
3	2	2	4	
1	1			

Answer: 3224

124 × 26 becomes

$$\begin{array}{r} 1 2 \\ 1 2 4 \\ 2 6 \times \\ \hline 7 4 4 \\ 2 4 8 0 \\ \hline 3 2 2 4 \\ \hline 1 1 \end{array}$$

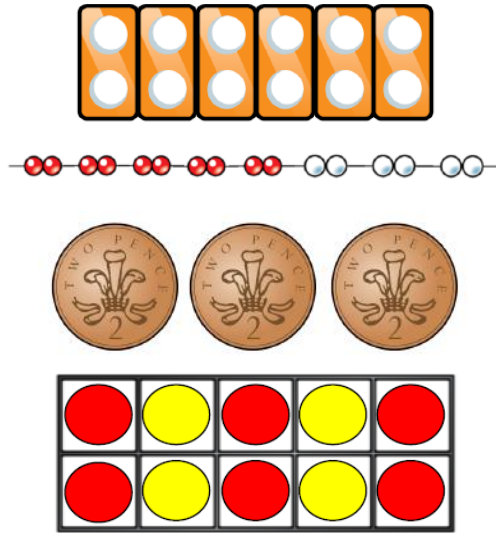
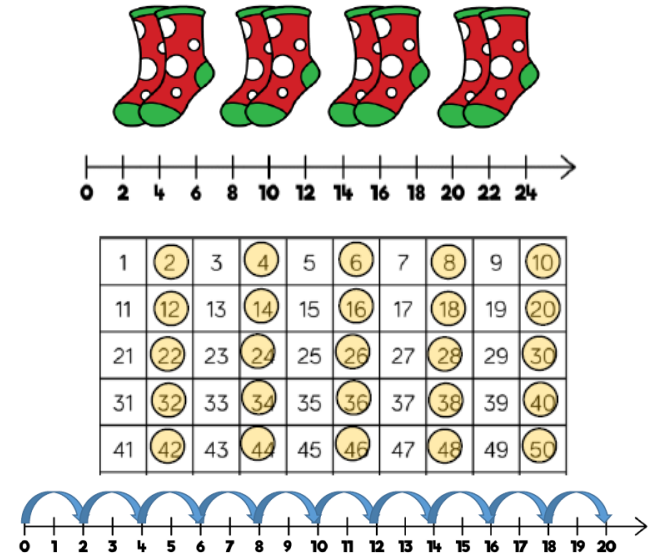
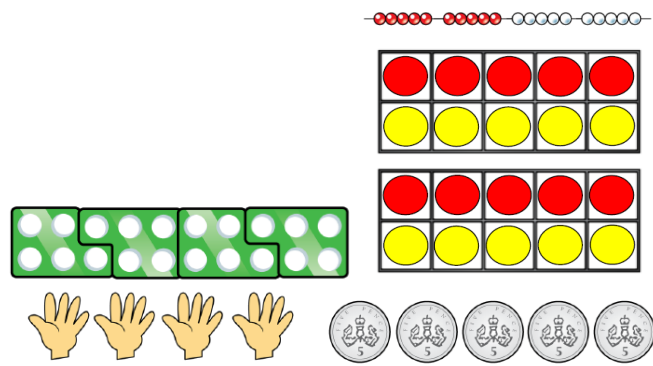
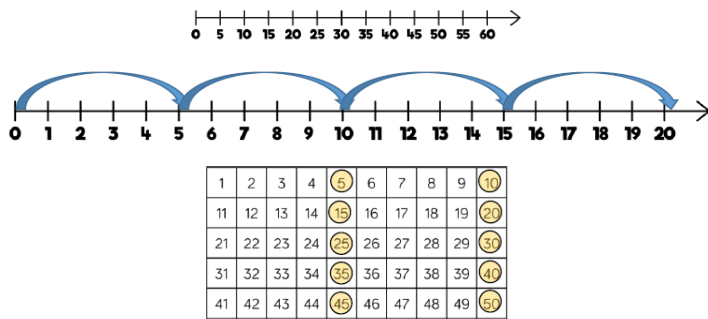
Answer: 3224

Multiplication -

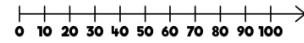
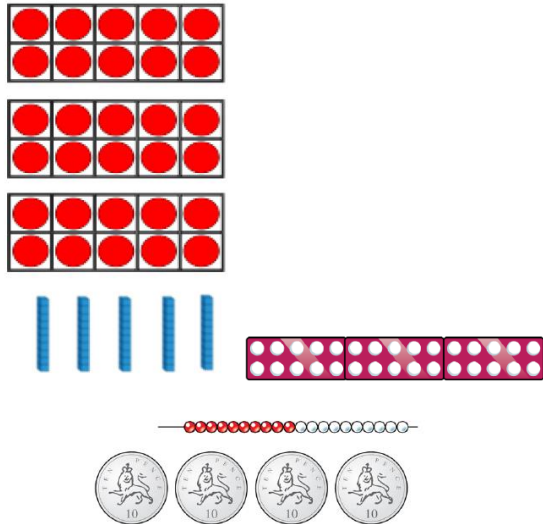
Times Tables

Years 2 - 6

Here are some models that represent each of the times tables. Use various models to develop fluency.

Skill	Concrete	Pictorial	Abstract
2 Times Table	 <p>Look for patterns of this times table using concrete manipulatives</p>		<ul style="list-style-type: none"> - Daily counting in multiples of 2, forwards and backwards. This can be supported using pictorial (number line, hundred square). - Notice how all the numbers are even - Notice there is a pattern in the ones.
5 Times Table			<ul style="list-style-type: none"> - Daily counting in multiples of 5, forwards and backwards. This can be supported using pictorial (number line, hundred square). - Notice the odds and evens pattern - Notice there is a pattern in the ones.

10
Times
Table



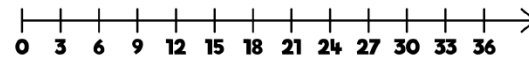
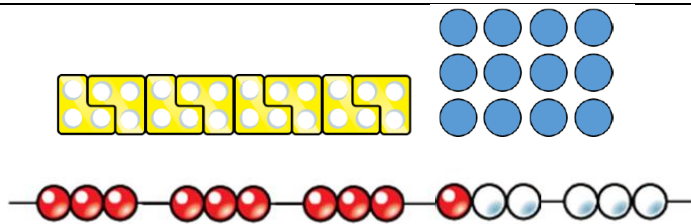
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Daily counting in multiples of 10, forwards and backwards. This can be supported using pictorial (number line, hundred square).

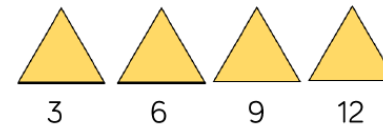
- Notice the tens increase by 1 ten each time

- Notice the pattern that all the multiples end in 0

3
Times
Table



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

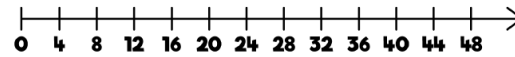
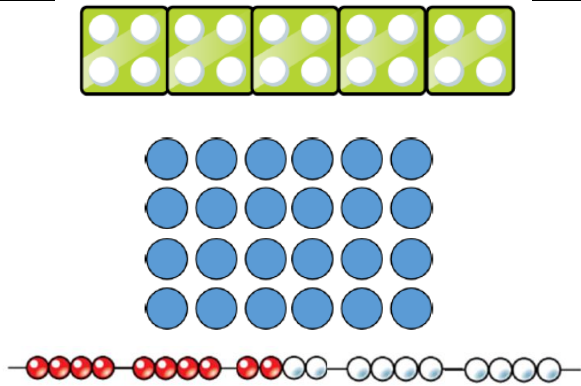


- Daily counting in multiples of 3, forwards and backwards. This can be supported using pictorial (number line, hundred square).

- Notice the odd, even, odd, even pattern

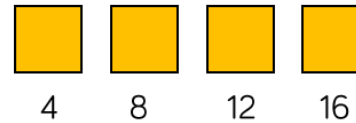
- Notice the pattern in the ones

4
Times
Table



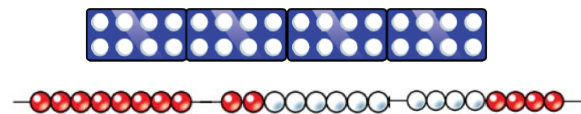
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21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

4	8	12	16	20
24	28	32	36	40
44	48	52	56	60



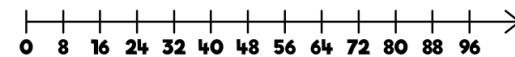
- Daily counting in multiples of 4, forwards and backwards. This can be supported using pictorial (number line, hundred square).
 - Make links to 2x table, noticing how each multiple is double the twos.
 - Notice all multiples are even
 - Notice the pattern in the ones within each group of 5 multiples

8
Times
Table

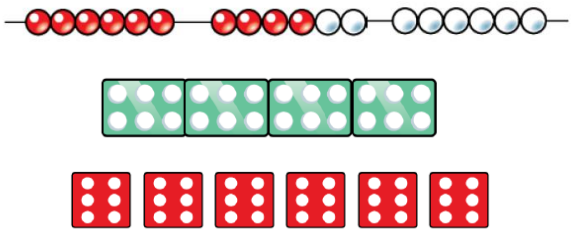
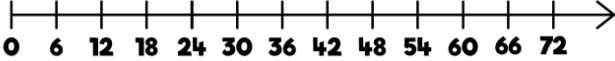
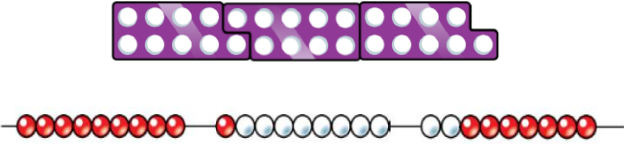
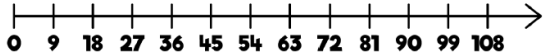


1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

8	16	24	32	40
48	56	64	72	80



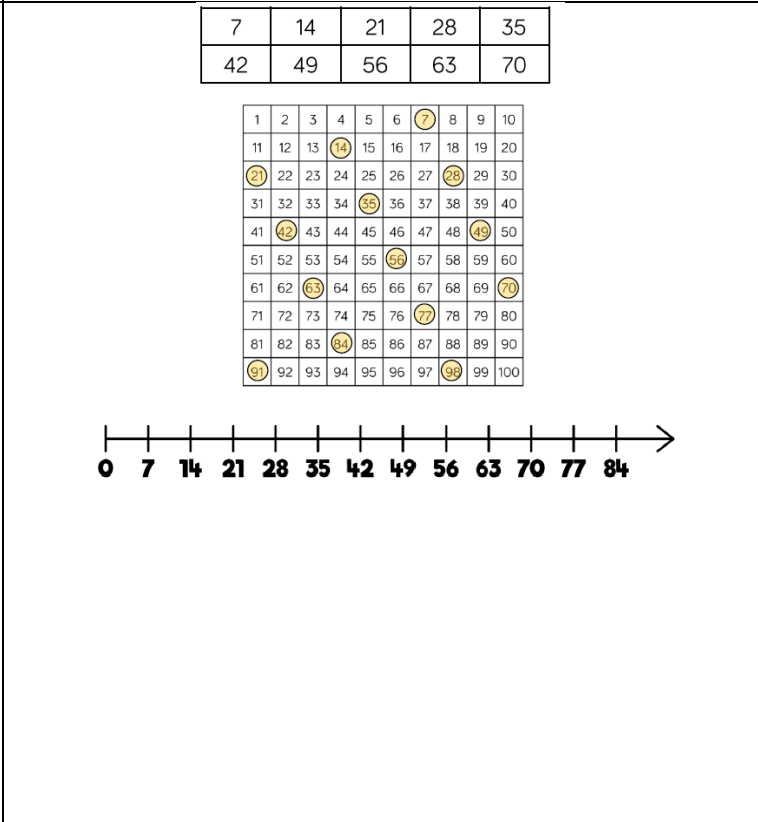
- Daily counting in multiples of 8, forwards and backwards. This can be supported using pictorial (number line, hundred square).
 - Make links to 4x table, noticing how each multiple is double the fours.
 - Notice all multiples are even

			<ul style="list-style-type: none"> - Notice the pattern in the ones within each group of 5 multiples 																																																																																																																			
<p>6 Times Table</p>		<table border="1" data-bbox="1137 272 1377 512"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr> <tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr> <tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td></tr> <tr><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td></tr> <tr><td>71</td><td>72</td><td>73</td><td>74</td><td>75</td><td>76</td><td>77</td><td>78</td><td>79</td><td>80</td></tr> <tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td><td>87</td><td>88</td><td>89</td><td>90</td></tr> <tr><td>91</td><td>92</td><td>93</td><td>94</td><td>95</td><td>96</td><td>97</td><td>98</td><td>99</td><td>100</td></tr> </table> <table border="1" data-bbox="1400 406 1713 512"> <tr><td>6</td><td>12</td><td>18</td><td>24</td><td>30</td></tr> <tr><td>36</td><td>42</td><td>48</td><td>54</td><td>60</td></tr> <tr><td>66</td><td>72</td><td>78</td><td>84</td><td>90</td></tr> </table> 	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	<ul style="list-style-type: none"> - Daily counting in multiples of 6, forwards and backwards. This can be supported using pictorial (number line, hundred square). - Make links to 3x table, noticing how each multiple is double the threes. - Notice all the multiples are even - Notice the pattern in the ones within each group of 5 multiples
1	2	3	4	5	6	7	8	9	10																																																																																																													
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<p>9 Times Table</p>		<table border="1" data-bbox="1115 1013 1355 1252"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr> <tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr> <tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td></tr> <tr><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td></tr> <tr><td>71</td><td>72</td><td>73</td><td>74</td><td>75</td><td>76</td><td>77</td><td>78</td><td>79</td><td>80</td></tr> <tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td><td>87</td><td>88</td><td>89</td><td>90</td></tr> <tr><td>91</td><td>92</td><td>93</td><td>94</td><td>95</td><td>96</td><td>97</td><td>98</td><td>99</td><td>100</td></tr> </table> <table border="1" data-bbox="1361 1177 1729 1257"> <tr><td>9</td><td>18</td><td>27</td><td>36</td><td>45</td></tr> <tr><td>54</td><td>63</td><td>72</td><td>81</td><td>90</td></tr> </table> 	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	9	18	27	36	45	54	63	72	81	90	<ul style="list-style-type: none"> - Daily counting in multiples of 9, forwards and backwards. This can be supported using pictorial (number line, hundred square). - Notice the pattern in the tens and 					
1	2	3	4	5	6	7	8	9	10																																																																																																													
11	12	13	14	15	16	17	18	19	20																																																																																																													
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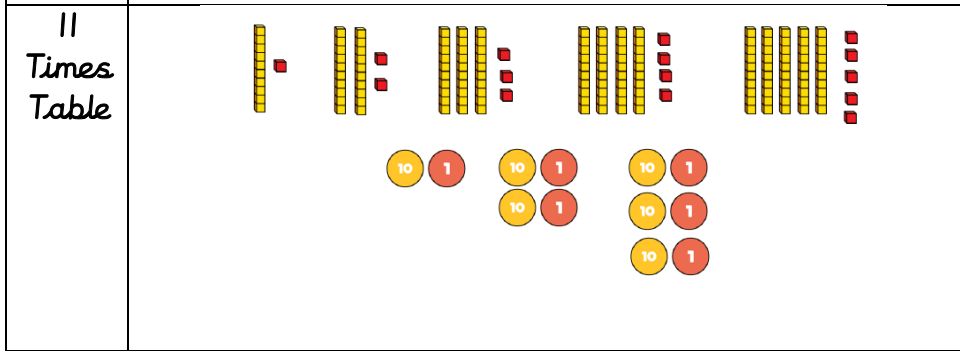
7
Times
Table

7	14	21	28	35
42	49	56	63	70

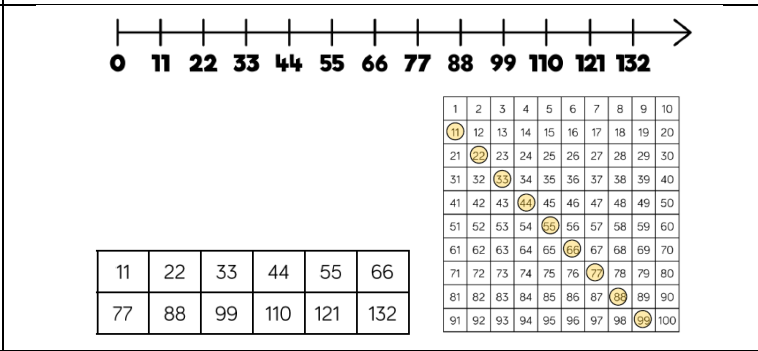


ones using the hundred square
- Notice the odd even pattern within the multiples

- Daily counting in multiples of 7, forwards and backwards. This can be supported using pictorial (number line, hundred square).
- This times table can be hard to learn due to the lack of obvious pattern in numbers, although children should know some facts due to commutativity
- Still notice odd, even pattern in multiples



11
Times
Table



- Daily counting in multiples of 11, forwards and backwards. This can be supported using pictorial (number line, hundred square).



12 Times Table

10 1 1

10 1 1

10 1 1

10 1 1



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

12	24	36	48	60
72	84	96	108	120
132	144			

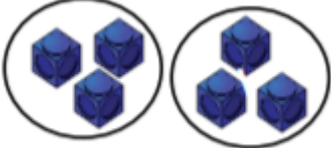


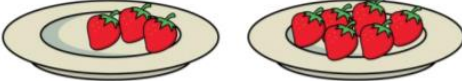
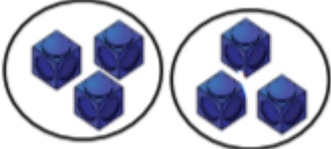

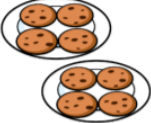
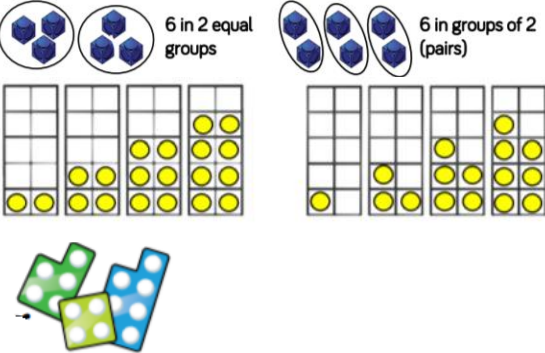

- Notice the pattern in the tens and ones
- Consider the pattern after 100

- Daily counting in multiples of 12, forwards and backwards. This can be supported using pictorial (number line, hundred square).
- Notice the pattern in the ones within each group of 5 multiples
- Make links to the 6x table and how each multiple is double the sixes

Division

EYES

Rather than having a key vocabulary section here, there are key questions listed in the 'Abstract' section for each objective to help children discuss their understanding.


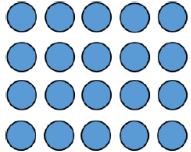
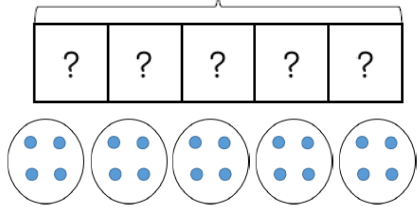
Skill	Concrete	Pictorial	Abstract
<p>To share items equally</p> <p>To check if items are shared equally</p>			<p>Can you share out these 6 cubes? How many will you put on each plate? Are they equal? How do you know? Can you share out 7 cubes equally?</p>
<p>To recognise equal groups</p>			<p>What groups can you see on these bead strings? Are the groups equal?</p>
<p>To make equal groups</p>		 <p>Are the groups equal?</p> 	<p>Can you put 2 crackers on each plate? Can you put 3 pencils in each pot? etc.</p>
<p>To recognise odd and even numbers to 10</p>		 <p>Prompt the children to recognise that when we share an odd number into 2 groups or into groups of 2, there is always 1 left over.</p> <p>Children can draw pictorial representations of odd and even numbers that they have looked at with the concrete resources.</p>	<p>Can you we group 5 children into pairs? Why not? What could we do to solve this problem? Can I always share a number into 2 groups? Can we get into pairs? Does that mean there are an odd or even number of players? If there are an odd number, how can this problem be solved?</p>

Year 1/2

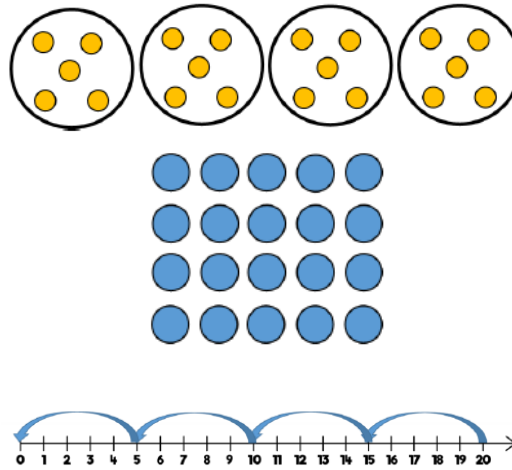
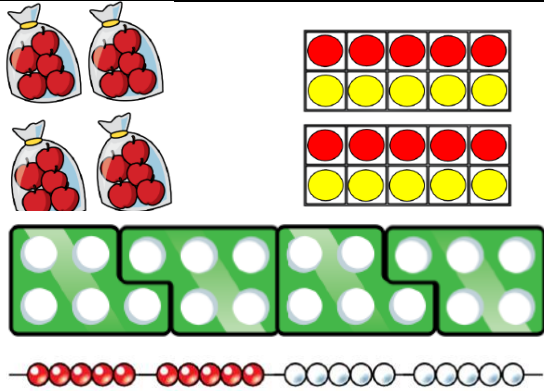
Key vocabulary

Divide, division, share, sharing, equal groups, unequal groups, sharing equally, ___ shared equally into ___ groups is ___, half, halving, odd, even, multiples of __, dividing, grouping, sharing, division, dividing, divide, divided by, divided into, division fact, grouping, sharing, share, share equally, left, left over, one each, two each, three each ... ten each, group in pairs, threes ... tens, equal groups of, halving

How can you tell if the groups are equal? How can you represent the equal groups? Do all numbers divide into equal groups? Can you sort ___ cubes into equal groups? Have I got equal groups? How do I know? Do I have any left over? Does each group need to be arranged in the same way to be equal? How can I share these objects equally?

Skill	Concrete	Pictorial	Abstract
<p>Solve one-step problems with division (sharing)</p>	<div style="text-align: center;">   </div> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>As infants, division (sharing) is shown as sharing amounts into equal groups. Year 2 is when the division symbol is introduced. There is no expectation that Year 1 record division formally i.e. a number sentence. They will work more concrete and pictorially to show multiplication as repeated addition.</p> </div>	<div style="text-align: center;"> <p>20</p>  </div>	<div style="border: 1px solid black; border-radius: 10px; padding: 10px; text-align: center;"> <p>There are 20 apples altogether. They are shared equally between 5 bags. How many apples are in each bag?</p> <p style="font-size: 2em; margin-top: 20px;">$20 \div 5 = 4$</p> </div>

Solve one-step problems with division (grouping)



There are 20 apples altogether.
They are put in bags of 5.
How many bags are there?

Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They can use concrete representations in fixed groups such as number shapes which helps to show the link between the multiplication and division.





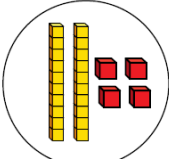
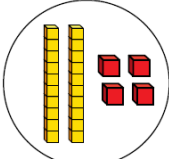
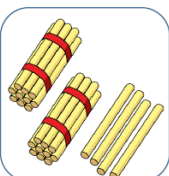
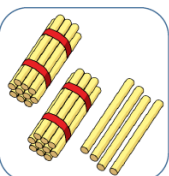




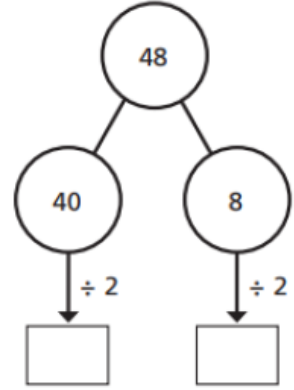




$$20 \div 5 = 4$$

Year 2/3

Key Vocabulary

Divide, division, share, sharing, equal groups, unequal groups, sharing equally, ___ shared equally into ___ groups is ___, half, halving, odd, even, multiples of __, dividing, grouping, sharing, division, dividing, divide, divided by, divided into, division fact, grouping, sharing, share, share equally, left, left over, one each, two each, three each ... ten each, group in pairs, threes ... tens, equal groups of, halving

How can you tell if the groups are equal? How can you represent the equal groups? Do all numbers divide into equal groups? Can you sort ___ cubes into equal groups? Have I got equal groups? How do I know? Do I have any left over? Does each group need to be arranged in the same way to be equal? How can I share these objects equally?

Skill	Concrete	Pictorial	Abstract						
<p>Divide 2-digits by 1-digit (sharing with no exchange)</p>	<table border="1" style="margin-bottom: 10px;"> <thead> <tr> <th style="background-color: #fff9c4;">Tens</th> <th style="background-color: #f8bbd0;">Ones</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">  </td> <td style="text-align: center;">  </td> </tr> <tr> <td style="text-align: center;">  </td> <td style="text-align: center;">  </td> </tr> </tbody> </table> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div>	Tens	Ones						<div style="border: 1px solid black; border-radius: 15px; padding: 10px; width: fit-content; margin: 0 auto;"> $48 \div 2 = 24$ </div> <div style="border: 1px solid black; padding: 10px; margin-top: 20px;"> <p>When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones. Straws, Base 10 and place value counters can all be used to share numbers into equal groups. Part-whole models can provide children with a clear written method that matches the concrete representation.</p> </div>
Tens	Ones								
									
									

Year 3/4

Key vocabulary

Division, dividing, divide, divided by, divided into, grouping, sharing, share, share equally, left, left over, remainder, one each, two each, three each ... ten each, group in pairs, threes ... tens

How do you know you've shared the objects equally? What's difference between sharing and grouping? Why would we partition 42 into 30 and 12 rather than 40 and 2? What do you notice about the partitioned number and the divisors? If I cannot share the tens equally, what do I need to do? How many ones will I have after exchanging the tens? How do we know 13 divided by 4 will have a remainder? Can we partition the number in more than one way to support dividing more efficiently? When dividing using a place value chart, in which direction do the digits move? What's the difference between sharing and grouping?

Skill	Concrete	Pictorial	Abstract
<p>Divide 2-digits by 1-digit (sharing with exchange)</p>			<p>$52 \div 4 = 13$</p> <p>$98 \div 7$ becomes</p> $\begin{array}{r} 14 \\ 7 \overline{) 98} \\ \underline{7} \\ 28 \\ \underline{28} \\ 0 \end{array}$ <p>Answer: 14</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones. Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows. Flexible partitioning in a part-whol model supports this method.</p> </div>

Year 3/4

Key vocabulary

Division, dividing, divide, divided by, divided into, division fact, halving, grouping, sharing, share, share equally, left, left over, remainder, one each, two each, three each ... ten each, group in pairs, threes ... tens, equal groups of, Inverse, square, squared cube, cubed

Why would we partition 42 into 30 and 12 rather than 40 and 2? What do you notice about the partitioned number and the divisors? If I cannot share the tens equally, what do I need to do? How many ones will I have after exchanging the tens? How do we know 13 divided by 4 will have a remainder? Can a remainder ever be more than the divisor? If we are dividing by 3, what is the highest remainder we can have? Can we partition the number in more than one way to support dividing more efficiently? When dividing using a place value chart, in which direction do the digits move? What's the difference between sharing and grouping?

Skill	Concrete	Pictorial	Abstract
Divide 2-digits by 1-digit (sharing with remainders)			<h3 style="margin: 0;">$53 \div 4 = 13 \text{ r}1$</h3> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> When dividing numbers with remainders, children can use Base 10 and place value counters to exchange one ten for ten ones. Starting with the equipment outside the place value grid will highlight remainders, as they will be left outside the grid once the equal groups have been made. Flexible partitioning in a part-whole model supports this method. </div>

Year 4

Key vocabulary

Division, dividing, divide, divided by, divided into, division fact, halving, grouping, sharing, share, share equally, left, left over, remainder, one each, two each, three each ... ten each, group in pairs, threes ... tens, equal groups of, Inverse, square, squared cube, cubed

Why would we partition 42 into 30 and 12 rather than 40 and 2? What do you notice about the partitioned number and the divisors? If I cannot share the tens equally, what do I need to do? How many ones will I have after exchanging the tens? How do we know 13 divided by 4 will have a remainder? Can a remainder ever be more than the divisor? If we are dividing by 3, what is the highest remainder we can have? Can we partition the number in more than one way to support dividing more efficiently? When dividing using a place value chart, in which direction do the digits move? What's the difference between sharing and grouping?

Skill	Concrete	Pictorial	Abstract
Divide 3-digits by 1-digit (sharing with exchange)		<p style="text-align: center;">844</p>	<p style="text-align: center; font-size: 24pt;">$844 \div 4 = 122$</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Children can continue to use place value counters to share 3-digit numbers into equal groups. Children should start with the equipment outside the place value grid before sharing the hundreds, tens and ones equally between the rows. This method can highlight remainders. Flexible partitioning in a part-whole model support this method.</p> </div>

Year 5

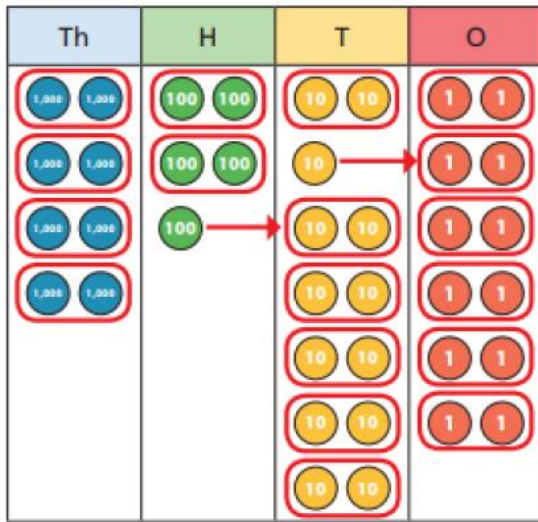
Division, dividing, divide, divided by, divided into, grouping, sharing, share, share equally, left, left over, remainder, one each, two each, three each ... ten each, group in pairs, threes ... tens, equal groups of, division fact, halving, Inverse, square, squared cube, cubed

Key vocabulary

In the hundreds column, how many groups of ___ are there in ___? In the thousands column, there are no groups of ___, what do we do? Why is the context of the problem so important when deciding how to round the remainders after a division? What happens to the digits? How are dividing by 10, 100 and 1000 related to each other? Why would we partition 42 into 30 and 12 rather than 40 and 2? What do you notice about the partitioned number and the divisors? If I cannot share the tens equally, what do I need to do? How many ones will I have after exchanging the tens? How do we know 13 divided by 4 will have a remainder? Can a remainder ever be more than the divisor? If we are dividing by 3, what is the highest remainder we can have? Can we partition the number in more than one way to support dividing more efficiently? When dividing using a place value chart, in which direction do the digits move? What's the difference between sharing and grouping?

Skill	Concrete	Pictorial	Abstract
Divide 3-digits by 1-digit (grouping)		<p>Children can continue to use grouping to support their understanding of short division when dividing a 3-digit number by a 1-digit number. Place value or plain counters can be used on a place value grid to support this understanding. Children can also draw their own counters and group them through a more pictorial method.</p>	<p style="text-align: center;">856 ÷ 4 = 214</p> <p style="text-align: center;">432 ÷ 5 becomes</p> $\begin{array}{r} 86 \text{ r}2 \\ 5 \overline{) 432} \end{array}$ <p style="text-align: center;">Answer: 86 remainder 2</p>

Divide 4-digits by 1-digit (grouping)



	4	2	6	6
2	8	5	13	12

$$8,532 \div 2 = 4,266$$

Place value or plain counters can be used on a place value grid to support children to divide 4-digits by 1-digit. Children can also draw their own counters and group them through a more pictorial method. Children should be encouraged to move away from the concrete and pictorial when dividing numbers with multiple exchanges.

Year 4/5

Key vocabulary

Division, dividing, divide, divided by, divided into, grouping, sharing, share, share equally, left, left over, remainder, one each, two each, three each ... ten each, group in pairs, threes ... tens, equal groups of, division fact, halving, Inverse, square, squared cube, cubed

What happens to the digits? How are dividing by 10, 100 and 1000 related to each other? Why would we partition 42 into 30 and 12 rather than 40 and 2? What do you notice about the partitioned number and the divisors? If I cannot share the tens equally, what do I need to do? How many ones will I have after exchanging the tens? How do we know 13 divided by 4 will have a remainder? Can a remainder ever be more than the divisor? If we are dividing by 3, what is the highest remainder we can have? Can we partition the number in more than one way to support dividing more efficiently? When dividing using a place value chart, in which direction do the digits move? What's the difference between sharing and grouping?

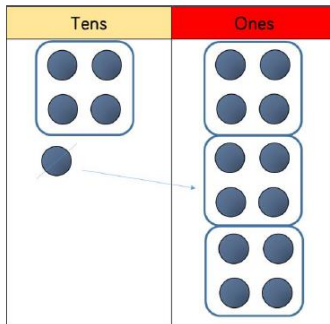
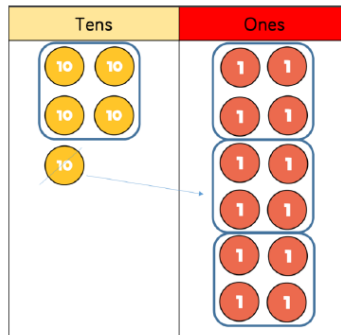
Skill

Concrete

Pictorial

Abstract

Divide 2-digits
by 1-digit
(grouping)



		1	3	
	4	5	12	

$$52 \div 4 = 13$$

When using the short division method, children use grouping. Starting with largest place value, they group by the divisor. Language is important here. Children should consider 'How many groups of 4 tens can we make?' and 'How many groups of 4 ones can we make?' Remainders can also be seen as they are left ungrouped.

Year 6

Key vocabulary

Division, dividing, divide, divided by, divided into, grouping, sharing, share, share equally, left, left over, remainder, one each, two each, three each ... ten each, group in pairs, threes ... tens, Equal groups of, division fact, halving, Inverse, square, squared cube, cubed

How can we use multiples to help us divide by a 2-digit number? In long division, what does the arrow represent (the movement of the next digit coming down to be divided)? What happens if we cannot divide the ones exactly by the divisor? How do we show what is left over? Does the remainder need to be rounded up or down?

Skill and key vocabulary	Concrete	Pictorial	Abstract																																			
<p>Divide multi-digits by 2-digits (short division)</p>	<div style="border: 1px solid black; padding: 5px;"> <p>When children begin to divide up to 4-digits by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective. Children write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.</p> </div>		<div style="text-align: center;"> <table border="1" style="margin: 0 auto; border-collapse: collapse;"> <tr><td></td><td></td><td>0</td><td>3</td><td>6</td></tr> <tr><td></td><td>12</td><td>4</td><td>4₃</td><td>7₂</td></tr> </table> <p>432 ÷ 12 = 36</p> <table border="1" style="margin: 0 auto; border-collapse: collapse;"> <tr><td></td><td>0</td><td>4</td><td>8</td><td>9</td></tr> <tr><td>15</td><td>7</td><td>7₃</td><td>13₃</td><td>13₅</td></tr> </table> <p>7,335 ÷ 15 = 489</p> <p>7,335 ÷ 15 = 489</p> <p>432 ÷ 12 = 36</p> <p>496 ÷ 11 becomes</p> <div style="text-align: center; margin: 10px 0;"> <table style="border-collapse: collapse;"> <tr><td></td><td></td><td style="text-align: center;">4</td><td style="text-align: center;">5</td><td style="text-align: center;">r1</td></tr> <tr><td></td><td></td><td colspan="3" style="border-top: 1px solid black; text-align: center;">5</td></tr> <tr><td style="border-right: 1px solid black; padding-right: 5px;">1</td><td style="border-right: 1px solid black; padding-right: 5px;">1</td><td style="padding: 0 5px;">4</td><td style="padding: 0 5px;">9</td><td style="padding: 0 5px;">6</td></tr> </table> </div> <p>Answer: $45\frac{1}{11}$</p> </div>			0	3	6		12	4	4 ₃	7 ₂		0	4	8	9	15	7	7 ₃	13 ₃	13 ₅			4	5	r1			5			1	1	4	9	6
		0	3	6																																		
	12	4	4 ₃	7 ₂																																		
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15	7	7 ₃	13 ₃	13 ₅																																		
		4	5	r1																																		
		5																																				
1	1	4	9	6																																		

Divide multi-digits by 2-digits (long division)

Children can also divide by 2-digit numbers using long division. They can write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.

When a remainder is left at the end of a calculation, children can either leave it as a remainder or convert it to a fraction. This will depend on the context of the question.

Children can also answer questions where the quotient needs to be rounded according to the context.

	0	4	8	9
15	7	3	3	5
-	6	0	0	0
	1	3	3	5
-	1	2	0	0
		1	3	5
-		1	3	5
				0

- 1 × 15 = 15
- 2 × 15 = 30
- 3 × 15 = 45
- 4 × 15 = 60
- 5 × 15 = 75
- 10 × 15 = 150

		0	3	6
1	2	4	3	2
	-	3	6	0
			7	2
	-		7	2
				0

- 12 × 1 = 12
- 12 × 2 = 24
- 12 × 3 = 36
- 12 × 4 = 48
- 12 × 5 = 60
- 12 × 6 = 72
- 12 × 7 = 84
- 12 × 8 = 96
- 12 × 7 = 108
- 12 × 10 = 120

432 ÷ 15 becomes

$$\begin{array}{r}
 28 \text{ r } 12 \\
 15 \overline{) 432} \\
 \underline{30 } \\
 132 \\
 \underline{132} \\
 0
 \end{array}$$

Answer: 28 remainder 12

432 ÷ 15 becomes

$$\begin{array}{r}
 28 \\
 15 \overline{) 432} \\
 \underline{30 } \quad 15 \times 20 \\
 132 \\
 \underline{120} \quad 15 \times 8 \\
 12
 \end{array}$$

$$\frac{12}{15} = \frac{4}{5}$$

Answer: 28 $\frac{4}{5}$

432 ÷ 15 becomes

$$\begin{array}{r}
 28.8 \\
 15 \overline{) 432.0} \\
 \underline{30 \downarrow} \\
 132 \\
 \underline{120 \downarrow} \\
 120 \\
 \underline{120} \\
 0
 \end{array}$$

Answer: 28.8

Appendix 1: Glossary

Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative - numbers can be added in any order.

Complement - in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference - the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange - Change a number or expression for another of an equal value.

Minuend - A quantity or number from which another is subtracted.

Partitioning - Splitting a number into its component parts.

Reduction - Subtraction as take away.

Subitise - Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.

Total - The aggregate or the sum found by addition.

Array – An ordered collection of counters, cubes or other item in rows and columns.

Commutative – Numbers can be multiplied in any order.

Dividend – In division, the number that is divided.

Divisor – In division, the number by which another is divided.

Exchange – Change a number or expression for another of an equal value.

Factor – A number that multiplies with another to make a product.

Multiplicand – In multiplication, a number to be multiplied by another.

Partitioning – Splitting a number into its component parts.

Product – The result of multiplying one number by another.

Quotient – The result of a division

Remainder – The amount left over after a division when the divisor is not a factor of the dividend.

Scaling – Enlarging or reducing a number by a given amount, called the scale factor

Appendix 2: Written method scripts

Column Addition Script -

No exchange example: $26 + 33 =$

Setting out

First, let's make 26. How many tens and ones do we need? We need 2 tens and 6 ones. Write T and O at the top of the columns. Write 2 in the tens column and 6 in the ones column. Which operation are we using? Addition. So now let's write the + sign at the right-hand side of the first number. What number are we adding? 33. Let's make 33. How many tens and ones do we need? We need 3 tens and 3 ones. This number will go underneath and stay in the columns. Write 3 tens in the tens column and 3 ones in the ones column. Write the equals lines underneath the numbers. The answer will go inside the lines.

Calculating - adding ones

Which column should we add first? The ones - we always add the ones first. Let's look in the ones column. How many ones are there? There are 6 ones and 3 ones. There are 9 ones in total. Do we have enough ones to make an exchange? No, why not? Because we don't have 10 or more ones. So the 9 is shown in the ones column inside the equal lines on our calculation.

Calculating - adding tens

Now let's move onto the tens column. How many tens are there? There are 2 tens and 3 tens. There are 5 tens in total. Do we have enough tens to make an exchange? No, why not? Because we don't have 10 or more tens. So the 5 is shown in the tens column, inside the equal lines on our calculation. So we can now see 26 add 33 is equal to 59.

Calculating - adding hundreds and beyond

Continue using the script as above but replacing the tens/ones wording with hundreds, thousands, etc. This is the same when calculating decimals so we still start at the right-hand column but now use the correct place value heading e.g. hundredths, tenths etc.

Column Addition Script -

Exchange example: $196 + 35 =$

Setting out

First, let's make 196. How many hundreds, tens and ones do we need? We need 1 hundred, 9 tens and 6 ones. Write H, T and O at the top of the columns. Write 1 in the hundreds column, 9 in the tens column and 6 in the ones column. Which operation are we using? Addition. So now let's write the + sign at the right-hand side of the first number. What number are we adding? 35. Let's make 35. How many tens and ones do we need? We need 3 tens and 5 ones. This number will go underneath and stay in the columns. Write 3 tens in the tens column and 5 ones in the ones column. Write the equals lines underneath the numbers. The answer will go inside the lines.

Calculating - adding ones

Which column should we add first? The ones - we always add the ones first. Let's look in the ones column. How many ones are there? There are 6 ones and 5 ones. There are 11 ones in total. Do we have enough ones to make an exchange? Yes. How do you know? Because we have 10 or more ones. So we can exchange 10 ones for 1 ten. Let's take the ten and give it to the tens. We show this by putting 1 ten under the tens column, below the equals sign. We then put the 1 one inside the equals sign.

Calculating - adding tens

Now let's move onto the tens column. How many tens are there? There are 9 tens, 3 tens and 1 ten (under the equals sign). There are 13 tens in total. Do we have enough tens to make an exchange? Yes. How do you know? Because we have 10 or more tens. So we can exchange 10 tens for 1 hundred. Let's take the hundred and give it to the hundreds. We show this by putting 1 hundred under the hundreds column, below the equals sign. We then put the 3 tens inside the equals sign in the tens column.

Calculating - adding hundreds

Now let's move onto the hundreds column. How many hundreds are there? There is 1 hundred and 1 hundred (under equals sign). There is 2 hundreds in total. Do we have enough hundreds to make an exchange? No, why not? Because we don't have 10 or more hundreds. So the 2 is shown in the hundreds column, inside the equal lines on our calculation. So we can now see 196 add 35 is equal to 231.

Calculating - adding thousands and beyond

Continue using the script as above but replacing the tens/ones wording with thousands, ten thousands etc. This is the same when calculating decimals so we still start at the right-hand column but now use the correct place value heading e.g. hundredths, tenths etc.

Column Subtraction Script -

No exchange example: $36 - 23 =$

Setting out

First, let's make 36. How many tens and ones do we need? We need 3 tens and 6 ones. Write T and O at the top of the columns. Write 3 in the tens column and 6 in the ones column. Which operation are we using? Subtraction. So now let's write the - sign at the right-hand side of the first number. What number are we subtracting? 23. Let's write 23. This number will go underneath and stay in the columns. Write 2 tens in the tens column and 3 ones in the ones column. Write the equals lines underneath the numbers. The answer will go inside the lines.

Calculating - subtracting ones

Which column should we look at first? The ones - we always look at the ones first. Let's look in the ones column. Do I have enough ones to subtract 3 ones or do I need to make an exchange? Yes, I have enough ones to subtract 3 ones from 6 ones so we don't need to make an exchange. So 6 ones take 3 ones is 3 ones. The 3 is shown in the ones column inside the equal lines on our calculation.

Calculating - subtracting tens

Now let's move onto the tens column. Do I have enough tens to subtract 2 tens or do I need to make an exchange? Yes I have enough tens to subtract 2 tens from 3 tens so we don't need to make an exchange. So 3 tens subtract 2 tens is 1 ten. So the 1 is shown in the tens column, inside the equal lines on our calculation. So we can now see 36 subtract 23 is equal to 13.

Calculating - adding hundreds and beyond

Continue using the script as above but replacing the tens/ones wording with hundreds, thousands, etc. This is the same when calculating decimals so we still start at the right-hand column but now use the correct place value heading e.g. hundredths, tenths etc.

Column Subtraction Script -

Exchange example: $804 - 352 =$

Setting out

First, let's make 804. How many hundred, tens and ones do we need? We need 8 hundreds, 0 tens and 4 ones. Write T and O at the top of the columns. Write 8 in the hundreds column, 0 in the tens column and 4 in the ones column. Which operation are we using? Subtraction. So now let's write the - sign at the right-hand side of the first number. What number are we subtracting? 352. Let's write 352. This number will go underneath and stay in the columns. Write 8 in the hundreds column, 0 in the tens column and 3 in the ones column. Write the equals lines underneath the numbers. The answer will go inside the lines.

Calculating - subtracting ones

Which column should we look at first? The ones - we always look at the ones first. Let's look in the ones column. Do I have enough ones to subtract 2 ones or do I need to make an exchange? Yes, I have enough ones to subtract 2 ones from 4 ones so we don't need to make an exchange. So 4 ones take 2 ones is 2 ones. The 2 is shown in the ones column inside the equal lines on our calculation.

Calculating - subtracting tens

Now let's move onto the tens column. Do I have enough tens to subtract 5 tens or do I need to make an exchange? No I don't have enough tens to subtract 5 tens from 0 tens so I need to make an exchange. So we exchange 1 hundred for 10 tens. Let's take the tens and give them to the tens. Now we have reduced the number of hundreds by 1 and increased the number of tens by 10. This is shown on the calculating by crossing out the 8 and writing a 7 beside it, then writing a 1 in the tens column beside the 0. Now can we subtract 5 tens from 10 tens? Yes, so 10 tens subtract 5 tens is 5 tens. We write the 5 in the tens column, inside the equal lines on our calculation.

Calculating - subtracting hundreds

Now let's move onto the hundreds column. Do I have enough hundreds to subtract 3 hundreds or do I need to make an exchange? Yes I have enough hundreds to subtract 3 hundreds from 7 hundreds so we don't need to make an exchange. So 7 hundreds subtract 3 hundreds is 4 hundreds. So the 4 is shown in the hundreds column, inside the equal lines on our calculation. So we can now see 804 subtract 352 is equal to 452.

Calculating - adding thousands and beyond

Continue using the script as above but replacing the tens/ones wording with thousands, ten thousands etc. This is the same when calculating decimals so we still start at the right-hand column but now use the correct place value heading e.g. hundredths, tenths etc.

Column Multiplication Script 2-digit number multiplied by a 1-digit number Example $23 \times 6 =$

Setting out

First, let's make 23. How many tens and ones do we need? We need 2 tens and 3 ones. Write H, T and O at the top of the columns. Write 2 in the tens column and 3 in the ones column. Which operation are we using? Multiplication. So now let's write the \times sign at the right-hand side of the first number. What number are we multiplying by? 6. This number will go underneath and stay in the columns. Write 6 ones in the ones column. Write the equals lines underneath the numbers. The answer will go inside the lines.

Calculating - multiplying ones

Which column should we multiply first? The ones - we always multiply the ones first. Let's look in the ones column. How many ones are there? There are 6 lots of 3 ones. There are 18 ones in total. Do we need to make an exchange? Yes. How do you know? Because we have 10 or more ones. So we can exchange 10 ones for 1 ten. Let's take the ten and give it to the tens. We show this by putting 1 ten under the tens column, below the equals sign. We then put the 8 ones inside the equals sign.

Calculating - multiplying tens

Now let's move onto the tens column. How many tens are there? There are 6 lots of 2 tens which is 12 tens. There is also 1 ten under the equals sign. There are 13 tens in total. Do we have enough tens to make an exchange? Yes. How do you know? Because we have 10 or more tens. So we can exchange 10 tens for 1 hundred. Let's take the hundred and give it to the hundreds. We show this by putting 1 hundred under the hundreds column, below the equals sign. We then put the 3 tens inside the equals sign in the tens column.

Calculating - multiplying hundreds

Now let's move onto the hundreds column. How many hundreds are there? There is 1 hundred (under equals sign). There is 1 hundred in total. Do we have enough hundreds to make an exchange? No, why not? Because we don't have 10 or more hundreds. So the 1 is shown in the hundreds column, inside the equal lines on our calculation. So we can now see 23 multiplied by 6 is equal to 138.

Calculating - 3+ digit numbers multiplied by 1-digit numbers

Continue using the script as above but replacing the tens/ones wording with thousands, ten thousands etc. This is the same when calculating decimals so we still start at the right-hand column but now use the correct place value heading e.g. hundredths, tenths etc.

Column Multiplication Script 2-digit number multiplied by a 2-digit number, Example $13 \times 15 =$

Setting out

First, let's make 13. How many tens and ones do we need? We need 1 tens and 3 ones. Write H, T and O at the top of the columns. Write 1 in the tens column and 3 in the ones column. Which operation are we using? Multiplication. So now let's write the \times sign at the right-hand side of the first number. What number are we multiplying by? 15. This number will go underneath and stay in the columns. Write 1 ten in the tens column and 5 ones in the ones column. Draw a line under the calculation and leave 2 line blank below it, before drawing the equals lines. This space is where the rest of the calculation will go and the answer will still go inside the equals lines.

Calculating - multiplying ones digit by top number

We are going to start by multiplying the top number by the ones digit from the bottom number. Which column should we multiply first? Let's look in the ones column. How many ones are there? There are 5 lots of 3 ones. There are 15 ones in total. Do we need to make an exchange? Yes. How do you know? Because we have 10 or more ones. So we can exchange 10 ones for 1 ten. Let's take the ten and give it to the tens. We show this by putting 1 ten under the tens column, directly below the equals sign. We then put the 5 ones in the ones column, directly below the calculation in the first line we left blank. Now let's look in the tens column. How many tens are there? There are 5 lots of 1 ten. There are 5 tens in total. Do we need to make an exchange? No, how do you know? Because we don't have 10 or more tens. Remember to add on the ten from below the equals lines at this point. So we have 5 tens plus 1 ten which gives us 6 tens. The 6 is shown in the tens column directly below the calculation in the first line we left blank. It's a good idea to cross out the ten we exchanged now it has already been added so we don't accidentally add it on in the next part of the calculation. We can now see 13 multiplied by 5 is 65.

Calculating - multiplying tens digit by top number

We are now going to multiply the top number by the tens digit from the bottom number. As we are no longer working with the ones, we put a 0 in the ones column in the second line we left blank. This is known as a place holder. First we will do 10 multiplied by the 3 ones. There are 10 lots of 3 ones which gives us 30 ones. We know this is the same as 3 tens. Do we need to make an exchange? No. How do you know? Because we do not have 10 or more tens. We put the 3 tens in the tens column in the second line we left blank. Now let's find 10 multiplied by 1 ten. There are 10 lots of 1 ten. There are 10 tens in total which gives us 100. How many hundreds are there? There is 1 hundred. Do we need to make an exchange? No, why not? Because there are not 10 or more hundreds. We write the 1 hundred in the hundred column in the second line we left blank. So we can now see 10 multiplied by 13 is 130.

Calculating - find the total

Now we have multiplied all the digits, we need to add the 2 parts of our calculation together. We do this using the formal written method for addition that we have already learned (see above script) to find the answer which will be written inside the equals lines.

Calculating - 3+ digit numbers multiplied by 2-digit numbers

Continue using the script above but replacing the tens/ones wording with thousands, ten thousands etc. As numbers get larger, ensure you multiply the top number by the ones digit of the bottom number first, recording the numbers below the calculation line as in the example above, just continuing to multiply each digit of the larger number i.e. ones by hundred, ones by thousand etc. Remember to cross out any additional digits you wrote as part of the exchanges as you go along once they have been included. Always write the 0 as a place holder once you move onto multiplying the top number by the tens digit of the bottom number.

Column Multiplication Script Decimal multiplied by a 1-digit number Example $6.25 \times 4 =$

Setting out

First, let's make 6.25. How many ones, tenths and hundredths do we need? We need 6 ones, 2 tenths and 5 hundredths. Write O, Tth and Hth at the top of the columns. Write 6 in the ones column, 2 in the tenths column and 5 in the hundredths column. Which operation are we using? Multiplication. So now let's write the \times sign at the right-hand side of the first number. What number are we multiplying by? 4. This number will go underneath the smallest place value column even though it does not match the place value headings above. Write the 4 in the hundredths column. Write the equals lines underneath the numbers. The answer will go inside the lines.

Calculating - multiplying hundredths

Which column should we multiply first? The smallest place value. Let's look in the hundredths column. How many hundredths are there? There are 4 lots of 5 hundredths. There are 20 hundredths in total. Do we need to make an exchange? Yes. How do you know? Because we have 10 or more hundredths. So we can exchange 20 hundredths for 2 tenths. Let's take the tenths and give it to the tenths. We show this by putting 2 tenths under the tenths column, below the equals sign. We then put the 0 hundredths inside the equals sign.

Calculating - multiplying tenths

Now let's move onto the tenths column. How many tenths are there? There are 4 lots of 2 tenths which is 8 tenths. There is also 2 tenths under the equals sign. There are 10 tenths in total. Do we have enough tenths to make an exchange? Yes. How do you know? Because we have 10 or more tenths. So we can exchange 10 tenths for 1 one. Let's take the tenths and give it to the ones. We show this by putting 1 one under the ones column, below the equals sign. We then put the 0 tenths inside the equals sign in the tenths column.

Calculating - multiplying ones

Now let's move onto the ones column. How many ones are there? There are 4 lots of 6 ones. There is also 1 one under the equals sign. There are 25 ones in total. Do we have enough ones to make an exchange? Yes. How do you know? Because we have 10 or more ones. So we can exchange 20 ones for 2 tens. Let's take the tens and give it to the tens. We show this by putting 2 under the tens column, below the equals sign. We then put the 5 ones inside the equals sign in the ones column.

Calculating tens

Now let's move onto the tens column. How many tens are there? There are 2 tens (under equals sign). There is 2 tens in total. Do we have enough tens to make an exchange? No, why not? Because we don't have 10 or more tens. So the 2 is shown in the tens column, inside the equal lines on our calculation. So we can now see 6.25 multiplied by 4 equals 25.

Calculating - 2+ digit numbers multiplied by 1-digit numbers

Continue using the script as above but replacing the tens/ones wording with hundreds, thousands, ten thousands etc. This is the same when calculating decimals so we still start at the right-hand column but now use the correct place value heading e.g. hundredths, tenths etc.

Short Division (without remainder) Script 2-digit number divided by a 1-digit number

Example $42 \div 3 =$

Setting out

First let's draw our division model (bus stop). The number we are dividing by (divisor) goes on the left hand side, outside the division model. The number we are dividing (dividend) goes on the right, inside the division model. So write 3 on the outside of the division model and 42 on the inside.

Calculating - dividing the tens column

Considering the tens column, how many groups of 3 tens can we make from our 4 tens? We can make 1 group of 3 tens with 1 ten remaining. We show this by writing the 1 group of tens above the tens column on the division model. What do you think we can do with this additional remaining ten? That's right, we can exchange it for ten ones. We show this by writing a small ten next to the ones digit, inside the division model. Now we have 12 ones.

Calculating - dividing the ones column

How many groups of 3 ones can we make with 12 ones? We can make 4 groups of 3 ones. We show this by writing the 4 groups above the ones column on the division model. So we can now see 42 divided by 3 is equal to 14 (the number on top of the division model).

Calculating - 3+ digit numbers divided by 1-digit numbers

Continue using the script as above but replacing the tens/ones wording with hundred, thousands, ten thousands etc and continuing until each digit of the dividend has been divided by the divisor.

Short Division (with remainder) Script 2-digit number divided by a 1-digit number

Example $93 \div 4 =$

Setting out

First let's draw our division model (bus stop). The number we are dividing by (divisor) goes on the left hand side, outside the division model. The number we are dividing (dividend) goes on the right, inside the division model. So write 4 on the outside of the division model and 93 on the inside.

Calculating - dividing the tens column

Considering the tens column, how many groups of 4 tens can we make from our 9 tens? We can make 2 groups of 4 tens with 1 ten remaining. We show this by writing the 2 groups of tens above the tens column on the division model. What do you think we can do with this additional remaining ten? That's right, we can exchange it for ten ones. We show this by writing a small ten next to the ones digit, inside the division model. Now we have 13 ones.

Calculating - dividing the ones column

How many groups of 4 ones can we make with 13 ones? We can make 3 groups of 4 ones with 1 one remaining. We show this by writing the 3 groups above the ones column on the division model and, beside that, write the remainder as r1 next to this. So we can now see 93 divided by 4 is equal to $23r1$ (the number on top of the division model).

Calculating - 3+ digit numbers divided by 1-digit numbers

Continue using the script as above but replacing the tens/ones wording with hundred, thousands, ten thousands etc and continuing until each digit of the dividend has been divided by the divisor.

Short Division (with remainder as decimal) Script 2-digit number divided by a 1-digit number

Example $93 \div 4 =$

Setting out

First let's draw our division model (bus stop). The number we are dividing by (divisor) goes on the left hand side, outside the division model. The number we are dividing (dividend) goes on the right, inside the division model. So write 4 on the outside of the division model and 93 on the inside. When using decimals, now add the decimal point and 00. (This is particularly useful for questions in the context of money or measures). Remember to add the decimal point to your answer as necessary too.

Calculating - dividing the tens column

Considering the tens column, how many groups of 4 tens can we make from our 9 tens? We can make 2 groups of 4 tens with 1 ten remaining. We show this by writing the 2 groups of tens above the tens column on the division model. What do you think we can do with these additional remaining tens? That's right, we can exchange it for ten ones. We show this by writing a small 1 next to the ones digit, inside the division model. Now we have 13 ones.

Calculating - dividing the ones column

Considering the ones column, how many groups of 4 ones can we make with 13 ones? We can make 3 groups of 4 ones with 1 one remaining. We show this by writing the 3 groups of ones above the ones column on the division model. What do you think we can do with this additional remaining one? That's right, we can exchange it for ten tenths. We show this by writing a small 1 next to the tenths digit, inside the division model. Now we have 10 tenths.

Calculating - dividing the tenths column

Considering the tenths column, how many groups of 4 tenths can we make with 10 tenths? We can make 2 groups of 4 tenths with 2 tenths remaining. We show this by writing the 2 groups of tenths above the tenths column on the division model. What do you think we can do with the 2 additional remaining tenths? That's right, we can exchange it for ten hundredths. We show this by writing a small 2 next to the hundredths digit, inside the division model. Now we have 20 hundredths.

Calculating - dividing the hundredths column

Considering the hundredths column, how many groups of 4 hundredths can we make with 20 hundredths? We can make 5 groups of 4 hundredths. We show this by writing the 5 groups of hundredths above the hundredths column on the division model. So we can now see 93 divided by 4 is equal to 23.25. Remember to add the unit of measure onto the answer e.g. £23.25 or 23.25m etc.

Calculating - 3+ digit numbers divided by 1-digit numbers

Continue using the script as above but replacing the tens/ones wording with hundred, thousands, ten thousands etc and continuing until each digit of the dividend has been divided by the divisor.

Short Division (with remainder as fraction) Script 2-digit number divided by a 1-digit number

Example $93 \div 4 =$

Setting out

First let's draw our division model (bus stop). The number we are dividing by (divisor) goes on the left hand side, outside the division model. The number we are dividing (dividend) goes on the right, inside the division model. So write 4 on the outside of the division model and 93 on the inside.

Calculating - dividing the tens column

Considering the tens column, how many groups of 4 tens can we make from our 9 tens? We can make 2 groups of 4 tens with 1 ten remaining. We show this by writing the 2 groups of tens above the tens column on the division model. What do you think we can do with this additional remaining ten? That's right, we can exchange it for ten ones. We show this by writing a small ten next to the ones digit, inside the division model. Now we have 13 ones.

Calculating - dividing the ones column

How many groups of 4 ones can we make with 13 ones? We can make 3 groups of 4 ones with 1 one remaining. We show this by writing the 3 groups above the ones column on the division model and, beside that, write the remainder as r1 next to this. So we can now see 93 divided by 4 is equal to 23r1 (the number on top of the division model).

Converting the remainder into a fraction

What number did we have as our remainder? r1. What number were we dividing by (divisor) in the original question? 4. So our remainder becomes the numerator and the divisor becomes the denominator. So our fraction is 1 over 4 which we know is read as one quarter. Do we need to cancel this fraction down any further? No. (If they do, they can do this in the way usually taught to cancel down fractions to the smallest fraction possible).

Calculating - 3+ digit numbers divided by 1-digit numbers

Continue using the script as above but replacing the tens/ones wording with hundred, thousands, ten thousands etc and continuing until each digit of the dividend has been divided by the divisor.

Long Division (with remainder as a whole number) Script 3-digit number divided by a 2-digit number Example $432 \div 15 =$

Setting out

First let's draw our division model (bus stop). The number we are dividing by (divisor) goes on the left-hand side, outside the division model. The number we are dividing (dividend) goes on the right, inside the division model. So write 15 on the outside of the division model and 432 on the inside. Remember to write the H, T and O at the top of the correct place value columns. These will help us to read the answer later but do not relate to some parts of the calculating that happens as we move through this method.

Calculating - dividing the hundreds column

Considering the hundreds column, how many groups of 15 can we take out of our 4? The answer is 0. If you want to, you can write a feint 0 in the answer space above the 4 or leave it blank.

Calculating - dividing the tens column

Now we look at the first 2 digits in our dividend as a whole number, in this case 43. How many groups of 15 can we take out of our 43? 2 groups. We show this on the calculation by writing a 2 above the division model above the 3. At the bottom of our calculation, we write the 30 (2 groups of 15) under the 43 and show this as a subtraction. So $43 - 30 = 13$.

Calculating - dividing the ones column

Now bring down the 2 from the original number (dividend) and put it next to the remaining 13. We can read this numbers as 132 now. How many groups of 15 can I take out of 132? 8 groups. We show this on the calculation by writing an 8 above the division model above the 2. At the bottom of our calculation, we write the 120 (8 groups of 15) under the 132 and show this as a subtraction. So $132 - 120 = 12$.

Calculating - finding the answer

Now we have used all the digits from the original number (dividend) in our calculation. As we can't take any more groups of 15 out of our previous number (12), this is our remainder. We write this next to the other digits that are above the division model as r12. This gives us the answer $28r12$

Note: This would be the same methods for 4+digit numbers. Also, it is the same as calculations that do not have a remainder except the last amount of groups would equal the number you are subtracting from e.g. $150-150=0$. If they end on 0, they know it divides equally with no remainder.

Long Division (with remainder as a fraction) Script 3-digit number divided by a 2-digit number Example $432 \div 15 =$

Setting out

First let's draw our division model (bus stop). The number we are dividing by (divisor) goes on the left-hand side, outside the division model. The number we are dividing (dividend) goes on the right, inside the division model. So write 15 on the outside of the division model and 432 on the inside. Remember to write the H, T and O at the top of the correct place value columns. These will help us to read the answer later but do not relate to some parts of the calculating that happens as we move through this method.

Calculating - dividing the hundreds column

Considering the hundreds column, how many groups of 15 can we take out of our 4? The answer is 0. If you want to, you can write a feint 0 in the answer space above the 4 or leave it blank.

Calculating - dividing the tens column

Now we look at the first 2 digits in our dividend as a whole number, in this case 43. How many groups of 15 can we take out of our 43? 2 groups. We show this on the calculation by writing a 2 above the division model above the 3. At the bottom of our calculation, we write the 30 (2 groups of 15) under the 43 and show this as a subtraction. So $43 - 30 = 13$.

Calculating - dividing the ones column

Now bring down the 2 from the original number (dividend) and put it next to the remaining 13. We can read this numbers as 132 now. How many groups of 15 can I take out of 132? 8 groups. We show this on the calculation by writing an 8 above the division model above the 2. At the bottom of our calculation, we write the 120 (8 groups of 15) under the 132 and show this as a subtraction. So $132 - 120 = 12$.

Calculating - finding the answer

Now we have used all the digits from the original number (dividend) in our calculation. As we can't take any more groups of 15 out of our previous number (12), this is our remainder. To show the remainder as a fraction, I need to write the remainder as the numerator and the number we were dividing by (divisor) as the denominator. If necessary, this fraction can be cancelled down further to the smallest equivalent fraction. So $12/15 = 4/5$. I can now write the remainder fraction next to the other digits above the division model as a fraction. This gives us the answer $28 \frac{4}{5}$.

Long Division (with remainder as a decimal) Script 3-digit number divided by a 2-digit number Example $432 \div 15 =$

Setting out

First let's draw our division model (bus stop). The number we are dividing by (divisor) goes on the left-hand side, outside the division model. The number we are dividing (dividend) goes on the right, inside the division model. So write 15 on the outside of the division model and 432 on the inside. Remember to write the H, T and O at the top of the correct place value columns. These will help us to read the answer later but do not relate to some parts of the calculating that happens as we move through this method.

Calculating - dividing the hundreds column

Considering the hundreds column, how many groups of 15 can we take out of our 4? The answer is 0. If you want to, you can write a feint 0 in the answer space above the 4 or leave it blank.

Calculating - dividing the tens column

Now we look at the first 2 digits in our dividend as a whole number, in this case 43. How many groups of 15 can we take out of our 43? 2 groups. We show this on the calculation by writing a 2 above the division model above the 3. At the bottom of our calculation, we write the 30 (2 groups of 15) under the 43 and show this as a subtraction. So $43 - 30 = 13$.

Calculating - dividing the ones column

Now bring down the 2 from the original number (dividend) next to the remaining 13. We can read this numbers as 132 now. How many groups of 15 can I take out of 132? 8 groups. We show this on the calculation by writing an 8 above the division model above the 2. At the bottom of our calculation, we write the 120 (8 groups of 15) under the 132 and show this as a subtraction. So $132 - 120 = 12$.

Calculating - dividing the tenths

Now we have used all the digits from the original number (dividend) in our calculation. As we can't take any more groups of 15 out of our previous number (12), this is our remainder. To show the remainder as a decimal, I need to put the decimal point in after the 2 on the original number (dividend) and 0 tenths. I also put a decimal point after the answer above the division model (quotient). I then bring the 0 down and put it next to the remaining 12. This now shows us 120. This is then divided by the original divisor, 15. So $120 \div 15 = 8$. This whole number is placed into the answer above the division model, in this case, above the 0 tenths we wrote on earlier. This whole number answer is then multiplied by the original divisor (15) to give us the final number we will subtract. $8 \times 15 = 120$. The 120 is written at the bottom of the calculation. We then do the final subtraction $120 - 120 = 0$. At this point, we have finished the calculation and can read our answer by looking at the number above the division model, 28.8 in this case.

Appendix 3: Additional Questions for General Mathematical Talk

How many different ways can you represent this calculation?	Which is the odd one out? Why?	Prove it/ Convince me.
What does each number in the calculation represent?	Can you show me using (any concrete manipulative)?	Can you draw this to show me pictorially?
Which method do you prefer?	Which method is the most efficient?	Can you calculate it mentally?
Is there more than one way we can solve this?	Is there more than one solution?	Can you use concrete apparatus to solve the problem?
Can you represent the problem in a picture?	Can you write a number sentence to show this?	Can you use any prior knowledge to check your answer?
What is the key vocabulary in this question?	Which operation(s) do we need to use?	What are the key bits of information?
Which strategy would you use and why?	How could you check your answer is correct?	How do we know if our answer is sensible?
How did you work it out?	What could we try next?	Explain your thinking/ method.
Describe/ Investigate the rule.		